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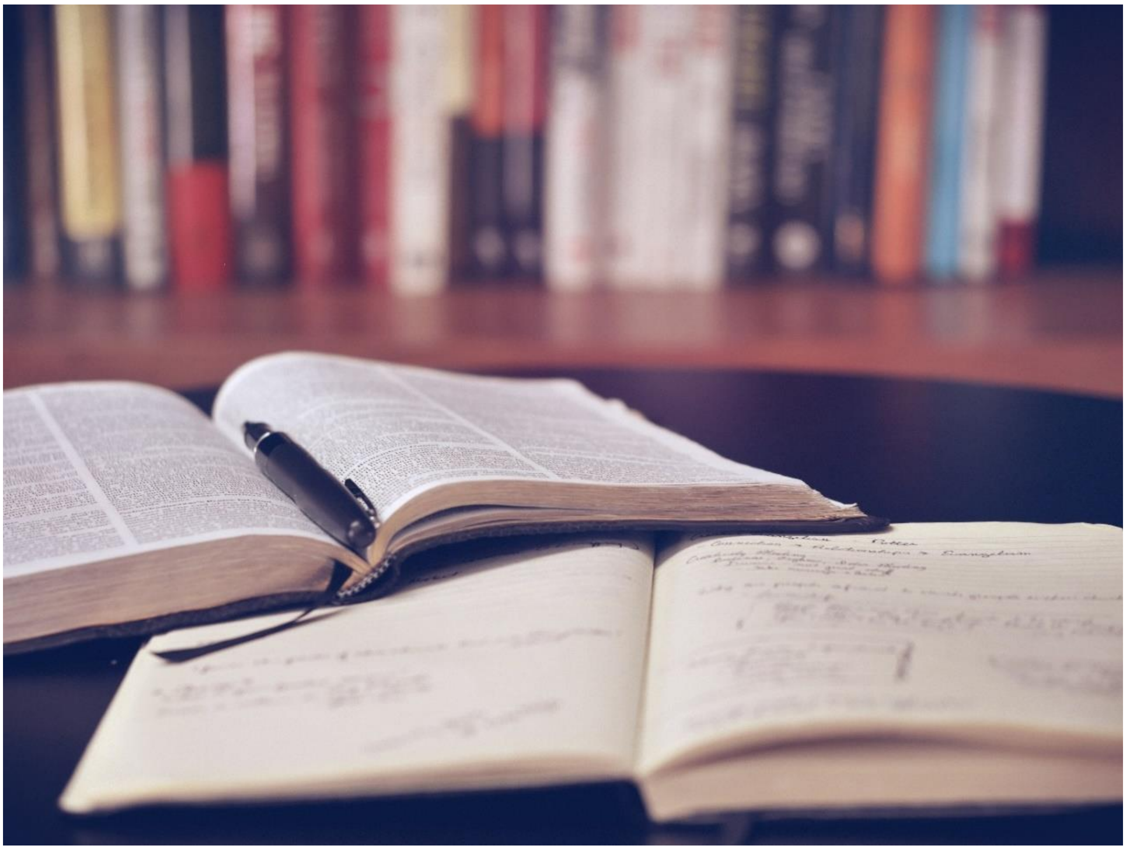

Time, space and hedonic prediction accuracy evidence from the Corsican apartment market

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Highlights

- A Bayesian hierarchical spatiotemporal model developed to study house prices
- A spatiotemporal random component gauging spatial and temporal correlation simultaneously
- Applying the INLA-SPDE approach to fit models
- Spatial variation and temporal trends mapped across the study area over time

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Abstract In this study, we propose a hedonic housing model to address spatial and temporal latent structures simultaneously.

With the development of spatial econometrics and spatial statistics, economists can now better assess the impact of spatial correlation on house prices. However, the simultaneous handling of spatial and temporal correlation is still under development.

Since spatial econometric models are limited to account for two kinds of correlation simultaneously, we propose using a hierarchical spatiotemporal model from spatial statistics. Based on a Bayesian framework and a stochastic partial differential equation (SPDE) approach, the estimation is carried out via INLA.

We then perform an empirical study on apartment transaction prices in Corsica (France) using the proposed model. The empirical results demonstrate that the prediction performance of the hierarchical spatiotemporal model is the best among all candidate models. Moreover, the hedonic housing estimates are affected by spatial effects and temporal effects. Ignoring these effects could result in serious forecasting issues.

Keywords Hierarchical spatiotemporal model · Hedonic price model · INLA-SPDE · Apartment market

JEL classifications: C11, C14, C33, R31

1 Introduction

It is widely agreed that housing locations affect housing prices. To explain this phenomenon, Can [14] identifies two effects: 1) "neighborhood effects", and 2) "adjacent effects". The first refers to sharing a series of location-specific amenities and public goods, whereas the second refers to a sort of spillover.

In addition to theoretical explanations, several economists have attempted to construct appropriate econometric models for evaluating properties. In particular, using hedonic price methods (HPMs), researchers can evaluate "neighbourhood effects" and interpret them as the marginal willingness to pay for corresponding attributes, even if these attributes are not measured directly. Nowadays, with the help of spatial econometrics [1], economists can also estimate "adjacent effects".

However, another argument emerges in the sense that researchers may omit the impact from temporal dimension if data are collected over time [19]. Specifically, the researchers have no information on repeated sales in most cases. As such, housing transaction data pertain to repeated cross-sections, which means that the data are composed of the different observations from given populations following chronological order.[1] To process such data, the most straightforward

[1] According to Dubé and Legros [19, p. 5], "The structure of real estate databases is different from conventional panels, or pseudo-panels, since the same observation is not necessarily repeated. Instead, real estate data are collection of many cross-sectional data pooled over time."

approach is to build a large pooled cross-section and apply pooled OLS regression with various space or time fixed dummy variables. However, this approach has an explicit limitation. It fails to capture the correlation in space and over time. As such, the estimated coefficients may be biased and predictions may be unrealistic.

Consequently, the aim of this article is threefold. A Bayesian hierarchical spatiotemporal model is initially introduced. Spatiotemporal correlation is gauged via the latent random effect component in the model. We illustrate how this model is fitted with two new techniques, namely integrated nested Laplace approximations (INLA) [39] and a stochastic partial differential equation (SPDE) approach [28]. INLA relies on direct numerical integration and is designed for latent Gaussian models. In addition, the SPDE approach makes use of Matérn covariance structures and Delaunay triangles to yield a Gaussian Markov random field (GMRF), which is a good approximation for the Gaussian random field (GRF). Lastly, a set of Bayesian hierarchical models are used for studying the Corsican apartment market. We compare the prediction accuracy of all candidate models, which involve spatial and temporal random components individually and jointly. A robust model is selected and is devoted to identifying the role of housing characteristics, geographical hotspots and to produce accurate market value predictions.

The study is structured as follows. In Section 2, we briefly review literature, in particular, certain new methods to gauge spatial and temporal correlation in property prices. In Section 3, we describe the hierarchical spatiotemporal model and briefly introduce the INLA method and the SPDE approach. In Section 4, we detail the data, empirical strategies and estimation results in our study. We conclude in Section 5.

2 Literature review: research trends in the HPM-based property valuation

Half a century ago, Lancaster [25] introduced his utility theory. He states that goods per se do not give utilities, but the involved characteristics provide utilities. As such, the utilities of a good equal the sum of the utilities of all characteristics. Rosen [38] integrates Lancaster theory into the market equilibrium framework and develops the hedonic price method.

In the context of house valuation, the hedonic price method ensures that a house buyer's utility is the function of the housing characteristics involved in the purchased house. Moreover, since the house buyer's utility can be expressed by the market equilibrium price of the purchased house [18], the house price is the function of the characteristics associated with the house. Following Malpezzi's definition [29], the empirical representation of a house price is given as:

$$P = f\left(S, N, L, C, T, \beta\right)$$

where P is the house price. S is the structural characteristics of the house, N represents the neighbourhood characteristics, L signifies the locational characteristics, C describes the contract conditions and T is time. β is the vector of the parameters to be estimated.

Although the hedonic price method provides the theoretical basis for investigating the impacts of housing characteristics on prices, the crucial challenge is the estimation. More specifically, this method provides a general analytical framework rather than guidance on a specified case [32]. Consequently, researchers must face the issues ranging from variable selection to model specifications.

To address these issues, we have observed some emerging trends [41] in the empirical studies on the hedonic price method, *e.g.,* applications of semi-parametric or non-parametric methods and applications of spatial econometrics [2].

The pioneering application of semi-parametric models on property valuation comes from Pace [33]. To address model specification problems, he suggests the so-called semi-parametric index regression. He also demonstrates how this model avoids misspecification and controls spatial trends.

Clapp [15] develops the local polynomial regression belonging to semi-parametric models. The model contains a nonlinear term based on latitudes and longitudes to calculate housing location values.

In the same year, Kammann and Wand [24] introduced the geoadditive models family, which is a mixture of additive modeling [12] and a geostatistical component. There are several candidate specifications for the geostatistical component, such as a kriging component or a smooth spatial trend component based on a tensor product of longitude and latitude.

Basile and his colleagues [9] apply a geoadditive model incorporating a 2-dimensional tensor product smoother for space to investigate European industrial locations. They clearly show that their model outperforms other parametric models in the sense that it allows to control unobserved spatial patterns, to reduce misspecification and to point out inward foreign direct investment clusters simultaneously. Further, it is possible to model space-time interactions via a 3-dimensional tensor product smoother [4] and therefore, the model incorporating such tensor product smoother can handle repeated cross-sections.

In short, the semi-parametric and geoadditive models have advantages such as flexibility, handling repeated cross-sections, avoiding model misspecification and hence, mitigating estimation biases.

Another trend relates to the widely used spatial econometrics [1]. Within the hedonic price framework, spatial econometric models is used for measuring location-specific amenities and "spillover" effects. However, applying spatial econometric models on repeated cross-sections is not rich in literature, most empirical studies focus on cross-sectional or panel data [1, 42].

After exploring the literature, we find two outstanding applications. Dubé and Legros [19] figure that the tools for analyzing geo-referenced house transaction data are very limited. Pooling data over time and applying time fixed dummy variables may lead to the biased estimates [34] because the dummy variables merely capture temporal variability, but neglect temporal correlation. Hence, they initially propose the so-called spatiotemporal autoregressive (STAR) model. Certainly, the STAR model is the extension of the corresponding spatial autoregressive (SAR) model. Authors replace the spatial weights matrix in a SAR model with a spatiotemporal weights matrix, so that spatial and temporal correlation is gauged. They show that the STAR model performs better than the SAR model in their case. However, due to its specification, the STAR model may have limited use.

Another approach called the unbalanced spatial lag pseudo-panel model with nested random effects comes from Baltagi et al. [7]. The authors are also confronted with a geo-referenced house transaction dataset, but they consider the hierarchical structure of the data. As a result, the data take the structure of a unbalanced pseudo-panel. Concerning the model, spatial correlation is captured by time-varying spatial weights matrices and temporal variability is captured by time fixed dummy variables. Nevertheless, temporal correlation is not considered in the model.

Notably, the above-mentioned models are estimated via the frequentist approach including maximum likelihood or restricted maximum likelihood (REML). However, regarding the uncertainty and model complexity, Bayesian inference may be a better choice [11, 31, 47]. To be more precise, the model parameters are considered as random variables in the Bayesian approach. Hence, both information in data and priori knowledge is absorbed into the posterior distribution of the model parameters.

Weller [47] holds the view that Markov Chain Monte Carlo (MCMC) methods have advantages over likelihood-based methods in fitting a linear mixed model because of considering the uncertainty of the interested parameters. According to Browne and Draper [11], both the MCMC and likelihood-based methods provide unbiased point estimates when they fit a two-level variance-components model with a large dataset. By contrast, when they fit a three-level logistics regression model, MCMC methods outperform likelihood-base methods in respect of the unbiased point estimates and coverage of interval estimates for random-effects variances.

Recent research [31] has suggested that INLA is an alternative of MCMC and REML methods in fitting a linear mixed model with multiple random components. INLA is remarkably efficient comparing with MCMC methods; whilst, it provides the accurate point estimates. According to another study [23], INLA-SPDE is an alternative of REML in fitting a linear mixed model including spatial random components. They indicate that INLA-SPDE can provide the robust estimates in the context of a small sample size. Additionally, it is likely that INLA-SPDE can handle the data followed a skewed normal distribution.

After investigating the advantages and drawbacks of the models mentioned above and the estimation, we will introduce the hierarchical spatiotemporal model, which is deemed as an extension of the geoadditive model. The fact is that we replace the kriging component in Kammann and Wand's model [24] with a spatiotemporal Gaussian Random Fields (GRFs) component so that we can gauge spatiotemporal random effects and repeated cross-sections simultaneously. It is believed that the hierarchical spatiotemporal model is more flexible than the spatial econometric models. Additionally, the estimation is done via the INLA-SPDE approach.

3 Method

3.1 Spatiotemporal data

Before reviewing the hierarchical spatiotemporal model, we will concentrate on data types. Palmquist [34] states that most HPM-based property valuing studies use micro-data. Dubé and Legros define the micro-data as

> "Observations that are points on a geographical projection...". [20, p. xi]

The definition implies that the geographical coordinates of properties are stored in the micro-data. From a geostatistical perspective, the micro-data pertain to the geostatistical data. It is a widely held view that such data can be considered as a stochastic process indexed on a continuous plane[2] [3, 46]. Further, regarding time dimension, the stochastic process can be indexed both in space and time.

Hence, we assume that $y\left(s_i, t\right)$ denotes the realization of the stochastic process, which describes the housing transaction price of house $i = 1, \ldots, n$ at location s_i with instant or span $t = 1, \ldots, T$.

3.2 Hierarchical models with space-time random effects

Bayesian hierarchical models are proposed finding housing price determinants and predicting house prices [36]. However, they have not been widely adopted due to high computational costs and complex estimation procedures. By contrast, hierarchical models can handle complex interactions via random components, which is determined by another regression model (see Lang et al. [27] for more details).

The proposed Bayesian hierarchical spatiotemporal model is derived from a modified ARHIER[3] model [40]. The key point is that this model involves a spatiotemporal random component defined by a first-order autoregressive (AR(1)) process in the next stage. More importantly, this spatiotemporal random component is devoted to capturing correlation in space and over time.

The first stage of the proposed model is written as

$$y(s_i, t) = z(s_i, t)\beta + \xi(s_i, t) + \epsilon(s_i, t) \tag{1}$$

[2] *E.g.,* a random field (RF)

[3] ARHIER stands for hierarchical autoregressive models.

where $z(s_i,t)$[4] is the vector of covariates referring to fixed effects and $\epsilon(s_i,t)$ is the measurement error following a Gaussian distribution. $\xi(s_i,t)$ is the space-time random component defined by a GRF varied over time. Thus, the second stage is used to depict the time-dependent GRF, which is assumed to be

$$\xi(s_i,t) = a\xi(s_i,t-1) + \omega(s_i,t) \tag{2}$$

where $\omega(s,t)$ is a time-independent GRF, whose covariance matrix is described by a Matérn covariance function.

$$Cov\left(\omega(s_i,t),\omega(s_j,t')\right) = \begin{cases} 0 & if\ t \neq t' \\ \sigma_\omega^2 C\left(\|s_i - s_j\|\right) & if\ t = t' \end{cases}$$

Where $C\left(\|s_i - s_j\|\right)$ denotes the Matérn correlation function, which depends on $\|s_j - s_i\|$.[5] $\|s_j - s_i\|$ is the Euclidean distance between the observation i and j. Notably, the Matérn correlation function implies that the spatial process is assumed to be second-stationary and isotropic [17]. Subsequently, the Matérn covariance function reads

$$\sigma_\omega^2 C\left(\|s_i - s_j\|\right) = \sigma_\omega^2 \times \frac{2^{1-\nu}}{\Gamma(\nu)} \times \left(\kappa \times \|s_i - s_j\|\right)^\nu \times K_\nu\left(\kappa \times \|s_i - s_j\|\right) \tag{3}$$

where Γ is the gamma function. K_ν is the modified Bessel function of the second kind with order ν. Generally, the ν is the smoothness parameter and is a non-negative number.[6] κ is the scaling parameter and is also a non-negative number. Based on the empirically derived definition [28], the relation among κ, ν and r is expressed as:

$$r = \frac{\sqrt{8\nu}}{\kappa}$$

where r indicates the distance where spatial correlation diminishes to 0.1.

Concerning the former part of In Eq. 2, a is the autoregressive parameter with $|a| < 1$. More importantly, to implement the model on software package R-INLA [30], we actually add a $\sqrt{1-a^2}$ term before $\omega(s,t)$ to ensure the stationary of the AR(1) process.

Since $\xi(s_i,t)$ represents the spatiotemporal GRF. Thus, its space-time separable covariance matrix of $\xi(s_i,t)$ is defined as,

$$\sum\nolimits_\xi = \sum\nolimits_T \bigotimes \sum\nolimits_S$$

where $\sum_T$ is the covariance matrix for the temporal process and $\sum_S$ is the Matérn covariance matrix for the spatial process. $\bigotimes$ is the Kronecker product. Therefore, we gauge the correlation in space and over time.

[4] $z(s_i,t) = \left(z_1(s_i,t), \ldots, z_p(s_i,t)\right)$
[5] $i \neq j$
[6] Based on Eq. 5, we have $\nu = \alpha - \frac{d}{2}$. In Eq. 3, d equals 2 and hence $\nu = 1$.

3.3 INLA-SPDE approach

Fitting the proposed Bayesian hierarchical spatiotemporal model is challenging, in particular, the “big n problem” [8] probably emerges.

Several solutions have been proposed to overcome these issues. Bakar and Sahu [5] develop the “spTimer” package[7], where they employ the MCMC approach with a low-rank approximation framework. A recent solution is to apply the INLA-SPDE approach. Apart from the advantages such as consideration uncertainty and low computational costs, the INLA-SPDE approach can estimates spatial range and other hyperparameters automatically. Further, the SPDE approach is devoted to computing complex spatiotemporal random effects and then the hierarchical models are fitted by INLA.

Let us now compute the spatial random component in the hierarchical models. Relying on the Matérn covariance function for the GRF and introducing a Markovian property, the values at each location will be conditional independent and then the dense Matérn covariance matrix will be substituted by a sparse precision matrix. To conclude, the GRF will be approximated by a GMRF to speed up the calculation. Currently, the difficulty becomes how to define that GMRF, which is the best substitution for the GRF, given local neighbourhood and the sparse precision matrix. Lindgren et al. [28] propose using the SPDE approach.[8] The numerical resolution of the SPDE given by the piecewise linear basis functions based on a mesh[9] can provide a good approximation to the Matérn covariance.

INLA will take over the following task. Introduced by Rue et al. [39], INLA produces posterior distribution of the interested parameters based on conditional probability rules and Laplace approximations (see Blangiardo and Cameletti [10] for more details).

4 An applied illustration using the Corsican apartment market

4.1 Background

Our study focuses on Corsica, one of the 18 French administrative regions, it is an island situated in the Mediterranean Sea.

As a “mountain in the sea”, one mountain range crosses the island from north to south, while flat areas or beaches are located at the edge of the island. Due to the particular mountainous topography, inhabitant activities are widely affected. On the island scale, the population distribution and population density express the significant spatial heterogeneity and spatial correlation.[10]

[7] According to the algorithm, the computational complexity of fitting the model reduces to $O(p^3)$ with $p \ll n$.

[8] Appendix 6.1 gives more details about the INLA-SPDE approach.

[9] We define a large number of non-overlapped triangles over the study area and the aggregate of the non-overlapped triangles is called a mesh.

[10] For instance, Bastia and Ajaccio are the capital cities of upper Corsica department and south Corsica department. 43% of the inhabitants live there. Other cities, such as

Furthermore, public services and infrastructure also display the strong spatial heterogeneity and spatial correlation. Hospitals, high schools and shopping malls are spatially centered in the cities, while villages lack public and private goods. This context may affect the local property prices.

4.2 Data

We use the dataset extracted from the "PERVAL" database. This database contains all information on property transactions, property attributes as well as buyers and sellers profiles. Additionally, most observations in the database are geo-referenced and therefore spatial analysis is used.

After removing the apartments with missing characteristics and eliminating the apartments corresponding to the tails[11] of the price distribution, there are 7634 observations spanning from 2006 to 2017.

The key variables of the dataset include the sale price, apartment structural characteristics and accessibility variables. The description of all variables is listed in Table 1.

The summary statistics of all continuous variables are presented in Table 2.

4.3 Empirical strategy

One objective of this study is to compare the prediction accuracy of different models that delineate the impacts of space and time in different ways. To meet this goal, we examine four models with different specifications. Starting from a linear additive model, we then add different random components, *e.g.,* a spatial random component, spatial and temporal random components jointly and a spatiotemporal random component. The summary of the random components involved in the hierarchical models is displayed in Table 3.

Let us explore these models in more detail. The dependent variable, the transaction price of an apartment in Euros, is transformed into a logarithmic scale. The application of the logarithmic transformation is not only intended to stabilize the variance of the dependent variable, but also to make its distribution approximately normal.

The base model (M0) is a linear additive model. You can find all apartment structural variables and accessibility variables in the model, but there are not any spatial or temporal random components.

Porto-Vecchio, Calvi, Corte, Ghisonaccia, L'Île-Rousse, Penta-di-Casinca and Propriano, are inhabited by 36% of the population [16].

[11] Since there are outliers, we exclude the observations whose prices are above 95% and below 5% of the price distribution.

Table 1 List of hedonic variables with description

Variable	Abbreviation	Description/Unit
STRUCTURAL ATTRIBUTES		
Room	RO	Number of rooms
Bathroom	BATH	Number of bathrooms
Garage	GAR	Number of garages
Floor	FL	Number of floors
Living area	SURF	Square meters (m^2)
Apartment type	AT	
Standard apartment (reference)	SA	
Duplex apartment	DU	
Studio apartment	ST	
Construction period	CP	
1850-1913 (reference)	PERIOD A	
1914-1947	PERIOD B	
1948-1969	PERIOD C	
1970-1980	PERIOD D	
1981-1991	PERIOD E	
1992-2000	PERIOD F	
2001-2010	PERIOD G	
2011-2017	PERIOD H	
ACCESSIBILITY VARIABLES		
Distance to the nearest beach	DBEAD	kilometres (km)
Distance to the nearest health facility	DHealFac	kilometres (km)
Distance to the nearest public primary school	DPuPriSch	kilometres (km)

Table 2 Descriptive statistics for hedonic housing prices in Corsica

	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Transaction Price	149467.08	58483.01	57445.76	100000	185347.95	325431.67
log(Transaction Price)	11.84	0.39	10.96	11.55	12.13	12.69
ROOM	2.672	0.967	0	2	3	8
BATHROOM	1.053	0.259	0	1	1	3
GAR	0.795	0.712	0	0	1	8
FLOOR	1.849	1.731	−3*	1	3	12
SURF	59.315	22.191	6	43	73	197
DBEAD	3.782	7.153	0.001	1.040	3.561	52.008
DHealFac	10.421	12.099	0.051	1.636	16.461	72.244
DPuPriSch	1.347	1.698	0.0001	0.469	1.544	39.513

N=7634; St. Dev.=standard deviation;
Pctl(25)=25% quantile; Pctl(75)=75% quantile;
∗The negative number appears because there are ”souplex” apartments.

Table 3 Summary of random components in the hierarchical models

Model Identifier	M0	M1	M2	M3
Spatial	None	$\xi(s_i)^*$	$\xi(s_i)^*$	None
Temporal	None	None	$\mu(t)$	None
Spatiotemporal	None	None	None	$\xi(s_i, t)$

* $\xi(s_i)$ is a time-independent GRF.

$$\begin{aligned} log(TransactionPrice) = & \beta_0 + \beta_1 RO + \beta_2 BATH + \beta_3 GAR + \beta_4 FL \\ & + \beta_5 AT + \beta_6 CP + \beta_7 DBEAD \\ & + \beta_8 DHealFac + \beta_9 DPuPriSch \\ & + f(SURF) \end{aligned}$$

where the living area is modeled as a nonlinear covariate regarding residuals and the literature [7, 43]. Concerning the nonlinear specification $f(\cdot)$, the default first-order random walk (RW1) smoother in R-INLA is used.

The second model (M1) is a combination of the first model (M0) with a spatial random component, also known as the time-independent GRF. This model is equivalent to the geoadditive model proposed by Kammann and Wand. In particular, the temporal dimension is collapsed to "zero thickness". In other words, time is still there, but it is expressed on a plane.

Based on M1, the third model (M2) involves an additional temporal random component $\mu(t)$. Notably, spatial and temporal effects are investigated jointly in this model. It is reasonable to consider the impacts of time on apartment sale prices even if we have already removed the effects of inflation. We intend to gauge the temporal correlation via an AR(1) process on the ordinal quarters and thus, M2 also formulates into a two-level hierarchical model.

$$\begin{aligned} &\mu(t) = N\left(0, \left(\tau\left(1-\rho^2\right)\right)^{-1}\right), \quad t = 1 \\ &\mu(t) = \rho \times \mu(t-1) + \varepsilon(t), \quad \varepsilon(t) \sim N\left(0, \tau_{ar1}^{-1}\right) \end{aligned} \tag{4}$$

where ρ is the autoregressive parameter and ε is the measurement error with precision τ_{ar1}.[12] In our case, there are 48 quarters spanning from the first quarter of 2006 to the fourth quarter of 2017.

Finally, M3 is the hierarchical spatiotemporal model introduced in subsection 3.2. It can be considered as an improvement of M1 in the sense that the time-independent GRF is replaced by a time-dependent GRF.

[12] *e.g.,* an inversion of the covariance matrix

Table 4 Priors for hyperparameters of the hierarchical model

Parameters in the models	Prior specification*
τ_{rw1} for precision of Surface	$(1, 0.5)$
Mean of spatial range, r	$(20, 0.8)$
σ_ω for spatial effect	$(0.4, 0.2)$
a for the AR(1) parameter in Eq.2	$(0.5, 0.7)$
τ_{ar1} for precision of AR(1) in Eq.4	$(5, 0.1)$
ρ for the AR(1) parameter in Eq.4	$(0.5, 0.7)$

* All priors are PC priors.

4.4 Implement details

Before running all candidate models, we highlight two key points, the mesh and prior. It is worth noting that the spatial covariance is evaluated using the SPDE approach. Through a triangular mesh, SPDE can define a GMRF to approximate the continuous GRF across the study area and hence, the discretely indexed GMRF reduces computational intensity.

For this study, we construct a non-convex hull mesh, as shown in Figure 1. Considering distances among housing locations and Corsican city sizes, we set the maximum triangle length to 10 km[13] in the interior domain, while the length is specified to 50 km in the outer extension to avoid boundary effects [6]. To reach a compromise between the dense housing locations in urban areas and too many tiny triangles, minimum distances among points are specified to 0.1 km. Lastly, we obtain the mesh containing 3117 vertices of triangles.

One advantage of Bayesian modeling is the inclusion of priori knowledge. In this study, we select the default penalized-complexity priors (PC priors) in INLA for all hyperparameters. A list of priors is shown in Table 4.

Concerning the RW(1) structure for the "living area" variable, we postulate that the prior follows $\text{Prob}(\tau_{rw1} > 1) = 0.5$.[14]

We assume the prior $\text{Prob}(r < 20) = 0.8$ for the mean posterior of the spatial range with units in kilometres. We also assume the prior $\text{Prob}(\sigma_\omega > 0.4) = 0.2$ for the mean posterior standard deviation of the spatial effects.

Concerning the AR(1) parameter in Eq.2, the prior is set to $\text{Prob}(a > 0.5) = 0.7$. We also posit the priors $\text{Prob}(\tau_{ar1} > 5) = 0.1$ and $\text{Prob}(\rho > 0.5) = 0.7$ in Eq.4.

Turning now on the model performance, the model assessment is based on the deviance information criterion (DIC) and the conditional predictive ordinate (CPO), which are directly obtained from R-INLA output [22].

[13] We convert the original map projection to the Universal Transverse Mercator (UTM) projection with 1-kilometre distance unit.

[14] Prob refers to the probability. τ_{rw1} is the precision parameter.

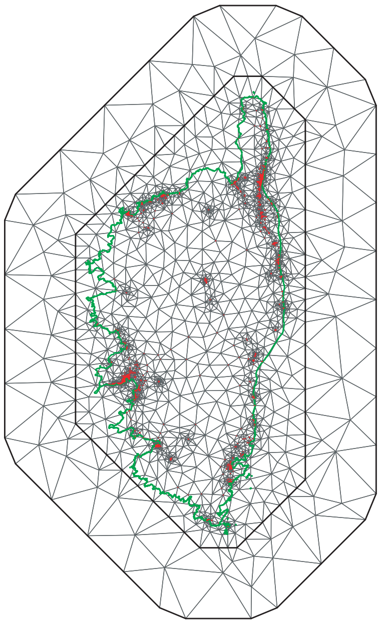

Fig. 1 The mesh. The boundary of Corsica is delineated in green and the sampled apartments are dotted in red. The inner black line distinguishes the inner mesh and the outer mesh.

DIC is a popular criterion to evaluate Bayesian hierarchical models proposed by Spiegelhalter et al. [45].

$$DIC = \overline{D} + p_D$$

Where $\overline{D}$ is the posterior mean of deviance of the model and p_D is the number of effective parameters in the model.

Introduced by Geisser [21], CPO returns a "leave one out" cross-validation score. More precisely, CPO calculates cross-validated predictive density at each observation. Roos and Held [37] suggest computing the mean logarithm CPO (LCPO) score, defined as

$$\text{LCPO} = -\frac{1}{N \times T} \sum_{t=1}^{T} \sum_{i=1}^{N} \log\left(\text{CPO}_{it}\right)$$

Lower DIC, LCPO scores indicate better-fitted models.

Table 5 Summary of model assessment

Model	DIC	LCPO	RMSE	Elapsed Time (Second)
M0	1,817.711	0.119	0.2636	20
M1	$-4,561.660$	-0.278	0.1849	73
M2	$-4,710.974$	-0.286	0.1848	114
M3	-5197.427	-0.304	0.1835	14769

Apart from the two criteria, model predictive power is also tested by randomly holding out 20% of the data. The training dataset contains the remaining 6107 observations. Subsequently, the candidate models predict the responses for the holdout dataset. The root mean squared error (RMSE) is considered to quantify prediction errors. More precisely, this criterion indicates the closeness between the predicted values and the observed values in the holdout dataset.

$$\text{RMSE} = \sqrt{\frac{1}{N \times T} \sum_{t=1}^{T} \sum_{i=1}^{N} \left(y\left(s_i, t\right) - \widehat{y\left(s_i, t\right)} \right)^2}$$

Lower RMSE values reflect better prediction.

Finally, analyses are carried out by using R 3.5.0 [35] and R-INLA 18.12.12 on a laptop equipped with an AMD Ryzen 5 1600 processor, 32G of RAM and Windows 10 operating system.

4.5 Results

Table 5 displays the predictive performance for each model. The running time of fitting each model varies from around 20 seconds to 4 hours 7 minutes.

Regarding the DIC scores, the models involving any random effect components (M1, M2, M3) outperform the referenced model (M0). The improvement of the DIC scores could be evidence of the usefulness of considering spatial and temporal correlation in modeling. Moreover, a possible reason for the well-fitted mixed models (M1, M2, M3) is that through random components, observations could borrow strength from their neighbours in space and over time. Typically, adding the spatial random component to the base model results in a considerable improvement in model fitting ($\Delta = -6379.371$). Additionally, the model including an additional temporal random component (M2) leads to a further improvement ($\Delta = -149.314$). M3 has the best goodness of fit in respect of the DIC values ($\Delta = -486.453$ relative to M2). The LCPO scores display in the same sequence as the DIC values. Further, to ensure the robustness of our findings, we apply different mesh designs[15] to the mixed effects models. The DIC and LCPO values together in the same sequence have been observed, irrespective of the different meshes.

Concerning the model predictive performance, the RMSE favours M3 as well. Therefore, among the candidate models, M3 is the best model to perform both estimation and prediction.[16]

For these reasons, M3 is selected as the final model. It is used to determine how various characteristics affect housing market values in the following section.

4.6 Inference based on the hierarchical spatiotemporal model

The posterior statistics, including the mean, 0.025 quantile and 0.975 quantile of the fixed effect coefficients for M3 are displayed in Table 6.[17]

In general, most covariates measuring the structural characteristics of apartments are significant with the expected signs, but several accessibility variables are not significant.

As expected, everything else remains the same, an additional room, bathroom, garage, floor and surface area on the logarithmic scale generally improve the apartment price. Moreover, the apartment built in the early year is likely cheaper than the recently built apartment.

For example, an additional room, bathroom, garage and floor could increase the expected apartment price by respectively 1.9% (95% $CI, 0.010; 0.029$)[18], 5.1% (95% $CI, 0.033; 0.069$), 4.9% (95% $CI, 0.040; 0.058$) and 2.0% (95% $CI, 0.017; 0.023$).

Typically, all things being equal, the duplex apartment is likely to be more expensive than the standard apartments with the gap of 3.4% (95% $CI, 0.009; 0.059$); whereas the studio apartment price could be 10.0% (95% $CI, -0.125; -0.075$) lower than the standard apartment price.

The expected price of the apartment built in the 1992-2000 period is 10.3% (95% $CI, 0.036; 0.169$) higher than the one built in the 1850-1913 period; Compared to the price of the apartment built in the 1850-1913 period, the expected price of the apartment built in the 2001-2010, 2011-2020 period would increase 22.1% (95% $CI, 0.154; 0.284$) and 22.4% (95% $CI, 0.169; 0.299$), respectively. It is believed that people prefer to live in the newly-built apartment rather than the older one.

According to the hedonic price theory, a positive coefficient for the living area would be expected. In Figure 2, although we observe an overall increasing trend over the whole range, there are some significant troughs and turning

[15] We try different meshes containing more tiny triangles. For all candidate models, the results are very close and all signs keep the same, but the computational time increase dramatically.

[16] Regression results of all candidate models are shown in Table 7.

[17] The prior and posterior distributions for the hyperparameters in M3 are displayed in Figure 4.

[18] CI stands for credible interval.

Table 6 Posterior estimates (mean and 95% interval) of the covariate coefficients in model M3

	mean	0.025 quant	0.975 quant
Intercept	11.733	11.637	11.830
ROOM	0.019	0.010	0.029
BATHROOM	0.051	0.033	0.069
GAR	0.049	0.040	0.058
FLOOR	0.020	0.017	0.023
DU	0.034	0.009	0.059
ST	−0.100	−0.125	−0.075
PERIOD B	−0.003	−0.066	0.060
PERIOD C	−0.018	−0.078	0.042
PERIOD D	0.018	−0.043	0.079
PERIOD E	0.038	−0.023	0.099
PERIOD F	0.103	0.036	0.169
PERIOD G	0.221	0.158	0.284
PERIOD H	0.224	0.161	0.287
DBEAD	−0.016	−0.021	−0.012
DHealFac	0.002	−0.005	0.010
DPuPriSch	−0.001	−0.003	0.001
SURF σ^2_{rw1}	0.033	0.019	0.054
σ^2_ϵ	0.025	0.024	0.026
σ^2_ω	0.108	0.090	0.125
Spatial Range (km)	1.484	1.291	1.698
AR(1)	0.991	0.988	0.994

points. A probable interpretation for these is given as follows. From 6 to approximately 20 m^2, there is a strong effect of decreasing marginal prices. The target clients for the small apartment would be low-income and young people, whose budget is relatively low. Additionally, the small apartment may be correlated with unattractive interior designs, which are not measured in our samples. Another remarkable turning point is 160 m^2. According to the statistic descriptive in Table 2, the upper range from 160 to 197 m^2 is in the tail of the samples. Furthermore, the large standard deviation may also be evidence of the turning point.

The proximity to the nearest beach has a significantly positive effect on the apartment price. If the distance between an apartment and its nearest beach increases by 1 km, the expected apartment price is estimated to decrease by 1.6% (95% CI, $-0.021; -0.011$). It is reasonable that the proximity to the nearest beach is capitalized into the apartment price [26].

Unexpectedly, insignificant relations are observed between the distance to the nearest public high school and apartment prices, between the distance to the nearest health facility and apartment prices. The insignificance of the

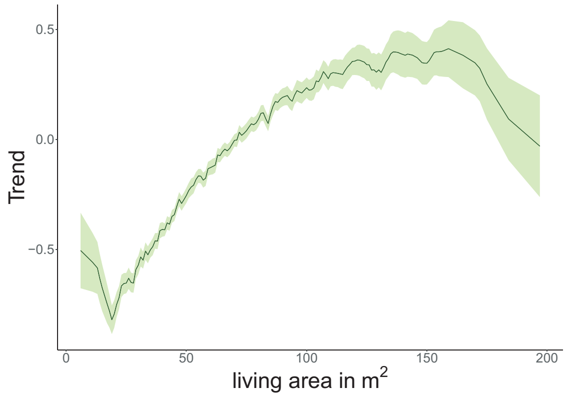

Fig. 2 Empirical mean and a 95% credible interval (the green area) of the living area

proximity to a public primary school may be because the local government provides perfect school bus services and some parents pick up their children from school. It is a widely held view that a public hospital is an amenity. However, according to our findings, it is likely that the closeness to a public hospital is not capitalized into the Corsican apartment price. A probable explanation is that a hospital may cause a large volume of traffic in the neighbouring areas. People living around may hear ambulance sirens every day. Additionally, in France, a patient usually makes an appointment to see his family medicine, rather than go to hospital directly.

In summary, the Corsican apartment price from 2006 to 2017 is largely determined by the structural and accessible characteristics.

As seen in the lower part of Table 6, we find that the majority of the remaining variance is due to the variance of the spatiotemporal process σ^2_ω, rather than the variance from the measurement error term σ^2_ϵ. Note that the estimate of σ^2_ω is over four times greater than the estimate of σ^2_ϵ. Again, this finding demonstrates that the Corsican apartment price is partly determined by the spatiotemporal structure.

For model M4, we find the mean posterior spatial range is 1.484 km (95% $CI, 1.291; 1.698$), meaning that the distance at which mean spatial correlation declines to 0.1 is 1.484 km. In other words, the spatial correlation decreases rapidly with distance. The mean posterior AR(1) coefficient is 0.991 with (95% $CI, 0.988; 0.994$). This indicates that the spatial random effects change rather slowly from quarter to quarter.

To further explore the spatiotemporal random effects ($\xi(s_i, t)$), we decided to plot the posterior mean of the spatial random fields in some quarters in Figure 3. This figure gives us the first impression that the spatial correlation of apartment prices across the island is relatively constant from one quarter to the next. It is thought that the constant spatial correlation over time clearly shows the high AR(1) coefficient.

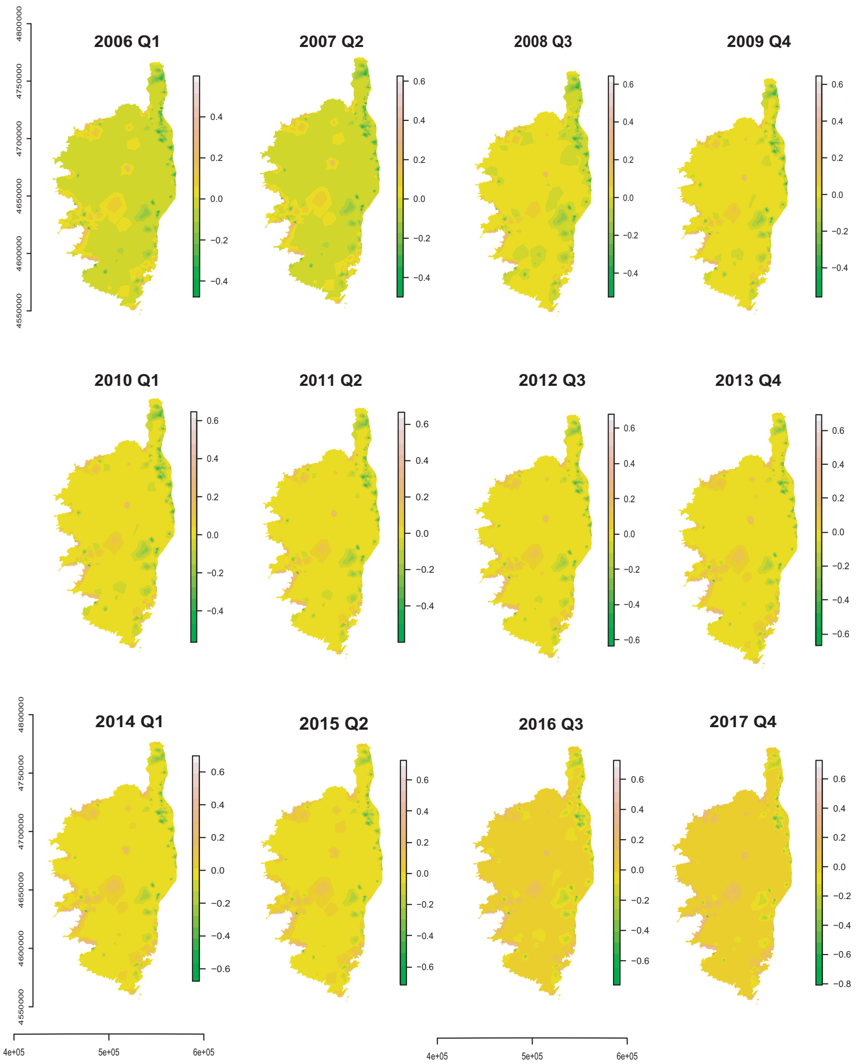

Fig. 3 Time-sliced plots displaying the posterior mean of spatiotemporal random effects. 2006 Q1 refers to the first quarter of the year 2006. We observe that the clusters near the island center change over time regarding the colour and surface.

We also observe several clusters. Inside these clusters, apartment prices are significantly affected by their location. In other words, the apartments located inside these clusters are spatially correlated in terms of prices. In general, most clusters are situated on the coastal plains. A few clusters are dotted in the junctions between the coastal plains and the inland area. More specifically, the cluster in which apartment prices are positively affected by their location is called a “hotspot”. Delving more deeply, if an apartment is located in a hotspot, the location will generate an additional increase on its price. In Figure 3, clusters in warm colours are hotspots. In contrast to the “hotspot”, the cluster in which apartment prices are negatively affected by their location is deemed a “cold spot”. In Figure 3, these clusters are plotted in cold colours. The “cold spots” include the eastern coastline and the whole northern tip of the island, whereas the “hotspots” include the northwestern area and the western coastline. Furthermore, the “cold spots” and “hotspots” near the coast evolve slowly over time. By contrast, the “hotspots” located in the inland area change relatively fast.

To gain further insight into the impact of location on apartment prices, we take an anti-logarithm, such that the exponential of the spatiotemporal random component works as a multiplication factor rather than an additional term in Eq.1.

Concerning the scale of Figure 3, it varies from -0.8 to 0.6 approximately, which means that in some “hotspots”, the location may increase the expected apartment price up to 82.21%[19]. However, in some “cold spot” zones, the location probably causes a 55.06%[20] reduction in the expected apartment price. This result clearly shows that the apartment location is a crucial factor in apartment price.

5 Conclusion

We propose using a powerful framework to study apartment prices. The framework relies on the flexible Bayesian hierarchical models and two novel data fitting techniques, INLA and SPDE.

To illustrate this framework, we investigate the Corsican apartment market using a unique dataset on the apartment transactions from 2006 to 2017.

We initially propose a set of the Bayesian hierarchal models incorporating spatial, temporal and spatiotemporal random components. All models are fitted by the recently-developed INLA and SPDE statistical approach. Fitting these models usually requires large computational resources and long running time, but the INLA SPDE approach addresses these problems by numerical integration and GMRFs. Thus, the approach provides fast and reliable Bayesian inference under affordable computing power. Additionally, through a model comparison, we demonstrate that the Bayesian hierarchical spatiotemporal model outperforms other mixed models and the linear additive model regarding the model predictive performance. The fact of such high performance is that observations borrow strength from their neighbours over space and time.

According to the results, it can be confirmed that most apartment attributes and the proximity to the nearest beach significantly affect the apartment price. In particular, we point out the nonlinear relation between the

[19] Note that if the logarithmic scale equals 0, there will be no impacts from the spatial random effects. Therefore, $exp^{(0.6)}$-1= 0.8221

[20] $exp^{(-0.8)}$-1=-0.5506

living area and apartment price. Additionally, regarding the spatial relations, we specify several “hotspots” and “cold spots”, where the apartment prices are significantly affected by their locations. Moreover, in general, the location of the “hotspots” and “cold spots” keeps stable over time. This phenomenon reflects that the mean posterior AR(1) value is high.

Currently, the INLA-SPDE approach is not widely seen in the hedonic pricing literature. We suggest that researchers consider both the Bayesian spatiotemporal hierarchical model and the INLA-SPDE approach as instruments in their toolbox when they investigate real estate economics and predict house prices.

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6 Appendix

6.1 SPDE

The expression of the SPDE in two dimensions is written as

$$\begin{gathered}\left(\kappa^2 - \mathit{\Delta}\right)^{\alpha/2} \times \xi\left(s\right) = W\left(s\right) \\ \alpha = \nu + \frac{d}{2},\ \kappa > 0,\ \nu > 0,\ s \in \mathbb{R}^2\end{gathered} \tag{5}$$

where $\mathit{\Delta}$ is the Laplace operator. α is an integer with the default setting of 2 in R-INLA. d indicates the dimension of the Euclidean space, such as $d = 2$ for classical planar space. $W\left(s\right)$ stands for the spatial Gaussian white noise process. The stationary solution of Eq. 5 is the GRF $\xi\left(s\right)$ with Matérn covariance function.

To implement the SPDE approach, Lindgren et al. [28] propose a two-step approach. At first, the entire study area is partitioned by a large number of non-overlapped triangles.[21] Then, based on the triangles, we apply a so-called piecewise linear approach [44].

$$\xi\left(s\right) = \textstyle\sum_{l=1}^{n} \psi_l\left(s\right) w_l \tag{6}$$

where $\xi\left(s\right)$ is a GRF. $\psi_l\left(x\right)$ is called the basis function. w_l are Gaussian-distributed weights. Therefore, given the approximation and conditional independence, we obtain a considerable computational gain, where the computational complexity dramatically reduces to $\mathrm{O}(n^{3/2})$ flops for GMRFs and $\mathrm{O}(n^2)$ for temporally correlated GMRFs [13].

[21] The process is also called triangulation and the aggregate of the non-overlapped triangles is called a mesh.

Table 7 Regression results of all candidate models (excluding M3)

	M0			M1			M2		
	mean	0.025quant	0.975quant	mean	0.025quant	0.975quant	mean	0.025quant	0.975quant
Intercept	11.693	11.595	11.791	11.735	11.635	11.836	11.704	11.586	11.822
ROOM	0.014	0.001	0.027	0.018	0.008	0.027	0.018	0.008	0.027
BATHROOM	0.087	0.060	0.113	0.056	0.037	0.074	0.055	0.037	0.073
GAR	0.016	0.006	0.026	0.043	0.035	0.052	0.049	0.040	0.057
FLOOR	0.009	0.005	0.013	0.020	0.017	0.023	0.020	0.017	0.023
DU	0.057	0.026	0.089	0.035	0.010	0.060	0.035	0.010	0.060
ST	-0.065	-0.100	-0.030	-0.100	-0.125	-0.075	-0.101	-0.125	-0.076
PERIOD B	0.094	0.002	0.185	0.003	-0.061	0.068	-0.001	-0.064	0.063
PERIOD C	0.081	-0.002	0.163	-0.009	-0.070	0.052	-0.014	-0.074	0.047
PERIOD D	0.158	0.076	0.239	0.029	-0.032	0.091	0.021	-0.040	0.082
PERIOD E	0.164	0.082	0.247	0.046	-0.016	0.108	0.041	-0.020	0.102
PERIOD F	0.205	0.116	0.294	0.120	0.054	0.187	0.116	0.049	0.182
PERIOD G	0.242	0.160	0.323	0.229	0.165	0.292	0.233	0.170	0.296
PERIOD H	0.239	0.158	0.320	0.232	0.169	0.296	0.217	0.154	0.280
DBEAD	-0.012	-0.013	-0.010	-0.017	-0.022	-0.012	-0.017	-0.022	-0.011
DHealFac	0.019	0.015	0.022	0.003	-0.005	0.011	0.003	-0.005	0.010
DPuPriSch	-0.004	-0.005	-0.004	-0.001	-0.004	0.001	-0.001	-0.004	0.001
SURF σ^2_{rw1}	0.048	0.028	0.076	0.034	0.019	0.056	0.034	0.019	0.056
σ^2_{ϵ}	0.073	0.071	0.076	0.029	0.028	0.030	0.029	0.028	0.030
σ^2_{ω}				0.109	0.092	0.130	0.114	0.100	0.134
Spatial Range (km)				1.571	1.360	1.840	1.549	1.332	1.783
ρ							0.944	0.845	0.991

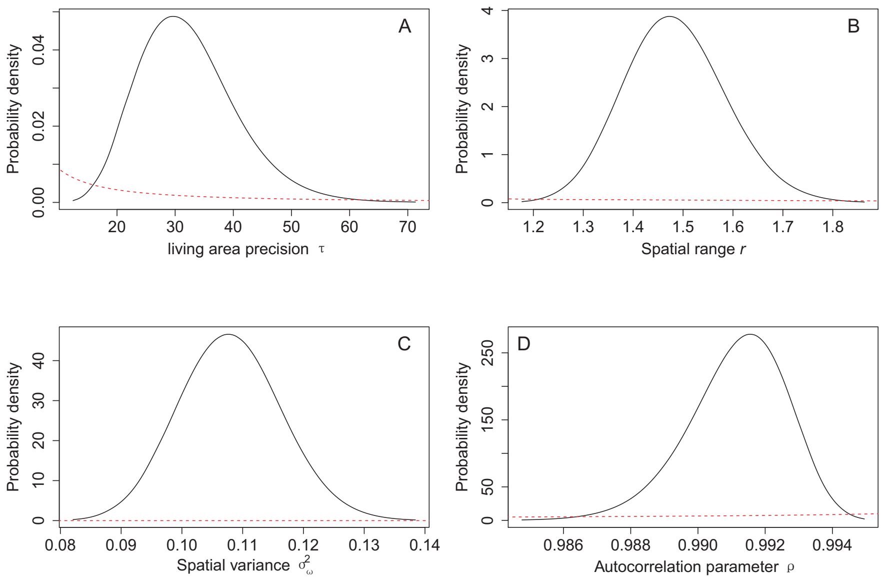

Fig. 4 Prior (red dash line) and posterior (solid line) distribution for hyperparameters in M3. A. Precision parameter τ_{rw1} for living area. B. Spatial range r in kilometres. C. Spatial variance σ_{ω}^{2} . D. Autocorrelation parameter a.

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