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Optimal bequests taxation in the steady state

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# Optimal bequests taxation in the steady state\*

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## Abstract

We consider an infinite-horizon economy populated by two types of individuals, some individuals being more productive than others. Individuals live one period and are altruistic toward their children. Assuming that the allocation received by a given individual depends only on his type and the one of his parent, we first determine the second-best steady state allocation and then study the optimal bequest and labor income tax functions, that are assumed to be *separable*. We first show that the second-best is not implementable with such tax schedules. We then demonstrate that it may be desirable to tax large bequests (and to subsidize low bequests), provided that individuals are sufficiently altruistic and the less productive individuals are sufficiently numerous. The taxation of large bequests is however not always part of the optimal solution. A numerical example suggests that no taxation of bequests is needed under moderate altruism, while large bequests should be subsidized when individuals are poorly altruistic.

Keywords: bequests, taxation, steady state.

JEL: H21, H24.

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# 1 Introduction

Despite representing a negligible part of fiscal revenue,<sup>1</sup> the taxation of bequests has always been subject to a heated controversy. Its opponents raise the concern that it is unbearable to tax individuals at death. Moreover, taxing bequests discourages labor supply, savings and destroy small businesses. Supporters of this tax argue that it allows to achieve equality of opportunity and has low efficiency costs.

The theoretical analysis of the optimal taxation of bequests raises a number of challenges. A first difficulty is that there is no clear empirical evidence about the bequest motive, the optimal level of bequests taxation being crucially dependent on this motive. Bequests could be driven by pure altruism (Barro (1974)). They can also be accidental due to the absence of perfect annuity markets. It could also be true that individuals derive utility from the mere fact of giving (Andreoni (1989)). Finally, it could be the outcome of a game between parents and children: parents exchange the promise of bequests with services provided by their children (Bernheim et al. (1985)). Presumably the decision to give results from a combination of these different motives and furthermore differs from an individual to another.

To illustrate the importance of the bequest motive, consider accidental bequests. In that case, taxation is not distortive and as a consequence bequests should be taxed heavily (Blumkin and Sadka (2004), Cremer et al. (2012)). In the pure altruism model on the other hand, taxation discourages individuals from leaving bequests and as such create distortions. We focus in this paper on this setting and ask what is the optimal bequest tax when individuals value the welfare of their offspring.

Earlier arguments in the literature build on the work by Chamley (1986) and Judd (1985) on capital income taxation, who show that capital should not be taxed in the long run. In the representative agent framework, bequests and savings are equivalent, implying that the Chamley-Judd result extends to the taxation of bequests (Cremer and Pestieau (2006)).

This result is valid when there is no heterogeneity among agents, meaning that only the efficiency role of taxes is taken into account. With individuals differing in productivity within each generation, the standard equity-efficiency trade-off appears. The first attempt to analyze this trade-off is due to Kaplow (2001). His analysis generated two main insights. First, when

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<sup>1</sup>In 2018, revenue of estate, inheritance and gift taxes represented less than 1% of GDP in all OECD countries (Drometer et al. (2018))

bequests are interpreted as a particular form of consumption, the Atkinson-Stiglitz theorem (Atkinson and Stiglitz (1976)) applies: when consumption and leisure are separable, there is no need to tax consumption. When applied to bequests, this suggests that they should not be taxed or subsidized. Second, the specificity of bequests is that they generate a positive externality from the donor to the recipient. As a consequence, bequests should be subsidized at the margin, acting like a Pigovian correction. Subsequent work has mainly developed these arguments (Kopczuk (2011), Cremer and Pestieau (2001), Farhi and Werning (2010)). These studies put two main restrictions on the model of the economy: either perfect correlation between the parents and the children is assumed (Kopczuk (2011)) or a two-period model is considered (Cremer and Pestieau (2001), Farhi and Werning (2010)).

Piketty and Saez (2013) developed a general model with individuals differing both in productivity and preferences. They analyze the steady state of this economy and consider the opportunity of taxing (linearly) bequests, when labor income is also subject to a linear tax. They find that it may be desirable to tax bequests, even with an optimally designed labor income tax system, and argue, using realistic simulations, that the marginal tax rate on bequests can be quite high. The main difference between Piketty and Saez (2013) and the previous articles is that the latter consider a bi-dimensional heterogeneity with individuals differing both with respect to their productivity and the amount of bequests received.

We also consider a model with such a bi-dimensional heterogeneity and analyze the optimal taxation of bequests when there exists an optimal non-linear tax system on labor income. Individuals live one period and have each one child. These individuals are altruistic and differ according to their productivity, which can be of two types, low and high. The productivities of parents and children are assumed to be uncorrelated.

We consider the steady state of this economy in which the allocation received by a given individual depends on his own type as well as the type of his parent. We first describe the second-best optimum. The optimal allocation is such that the child of a highly productive individual should be better-off than the one of a low productivity individual with the same productivity level. This allows to relax the incentive constraint of the parents, a result that has been put forward by Farhi and Werning (2010).

We then study the optimal design of *separable* labor income and bequests nonlinear tax functions. We show that these tax instruments do not allow to implement the optimum. As

for the design of the bequests tax, we find that it may be optimal to redistribute from high to low bequests. A sufficient condition for this result is that individuals are sufficiently altruistic and that the less productive individuals are sufficiently numerous. Finally, we exhibit a numerical example in which the redistribution of bequests takes place the other way around when individuals have a limited altruism motive.

## 2 The economy

Individuals live one period and differ by the level of their productivity (type), which is assumed to be private information. In each period, there are  $N^L$  individuals with productivity  $\omega^L$  and  $N^H$  individuals with productivity  $\omega^H$ ,  $\omega^L < \omega^H$ . We consider successive generations: each generation has measure 1, lives one period, and is replaced by the next generation.

We assume a utility function additively separable between leisure and consumption. Note that the allocation received by a given individual may depend on his own productivity but also on the productivity of his ancestors. *We assume the allocation of a given individual only depends on his productivity and the one of his parent.* With this restriction, the preferences of an individual with productivity  $i$  living in period  $t$  with a parent of productivity  $j$  are given by:

$$V_t^{ij} \equiv U(c_t^{ij}, l_t^{ij}) + \gamma \sum_k p^{ki} V_{t+1}^{ki}, \quad (1)$$

where  $U(c_t, l_t) = u(c_t) - v(l_t)$ ,  $c_t$  is consumption at date  $t$  and  $l_t$  labor supply. The probability that a child is of type  $k$  when his parent is of type  $i$  is  $p^{ki}$ . We assume *no correlation* between the types of the parents and the children, so that:  $p^{HH} = p^{HL} \equiv p^H = N^H/N$  and  $p^{LH} = p^{LL} \equiv p^L = N^L/N$ . The parameter  $0 \leq \gamma < 1$  represents the degree of altruism. It is assumed to be identical for all individuals.

## 3 Optimal steady state allocation

We consider the steady state of this economy: the distribution of consumptions and labor supplies is the same in every period.

The social planner maximizes the utilitarian welfare of a representative generation. It is shown in the appendix that his program simplifies to:

$$\begin{aligned} & \max N^L p^L U(c^{LL}, y^{LL}/\omega^L) + N^H p^L U(c^{LH}, y^{LH}/\omega^L) \\ & + N^L p^H U(c^{HL}, y^{HL}/\omega^H) + N^H p^H U(c^{HH}, y^{HH}/\omega^H) \end{aligned}$$

st

$$N^L p^L (y^{LL} - c^{LL}) + N^H p^L (y^{LH} - c^{LH}) + N^L p^H (y^{HL} - c^{HL}) + N^H p^H (y^{HH} - c^{HH}) \geq 0,$$

and

$$\begin{aligned} U(c^{HH}, y^{HH}/\omega^H) + \gamma(p^L V^{LH} + p^H V^{HH}) &\geq U(c^{LH}, y^{LH}/\omega^H) + \gamma(p^L V^{LL} + p^H V^{HL}) \\ U(c^{HL}, y^{HL}/\omega^H) + \gamma(p^L V^{LH} + p^H V^{HH}) &\geq U(c^{LL}, y^{LL}/\omega^H) + \gamma(p^L V^{LL} + p^H V^{HL}), \end{aligned}$$

where  $y = \omega^j l$  is the production of a type  $j$  individual. The second constraint is the resource constraint in each period: total consumption should not exceed total production. Bequests do not appear in this equation because inheritances received and bequests left exactly cancel out in the aggregate. The second group of constraints represent incentive constraints: a type  $j$  individual should not want to pretend that he is of type  $i$ . As usual in the optimal taxation literature (Stiglitz (1987)), only the constraints from the high to the low types bind at the optimum. This is checked in proposition 1.

We now solve this program. From (1), we have:

$$\begin{aligned} V^{LL} &= U(c^{LL}, y^{LL}/\omega^L) + \gamma(p^L V^{LL} + p^H V^{HL}) \\ V^{LH} &= U(c^{LH}, y^{LH}/\omega^L) + \gamma(p^L V^{LL} + p^H V^{HL}) \\ V^{HL} &= U(c^{HL}, y^{HL}/\omega^H) + \gamma(p^L V^{LH} + p^H V^{HH}) \\ V^{HH} &= U(c^{HH}, y^{HH}/\omega^H) + \gamma(p^L V^{LH} + p^H V^{HH}), \end{aligned}$$

implying:

$$\begin{aligned} V^{LH} - V^{LL} &= U(c^{LH}, y^{LH}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L) \\ V^{HH} - V^{HL} &= U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H). \end{aligned}$$

The incentive constraints can thus be re-written in the following way:

$$\begin{aligned} &U(c^{HH}, y^{HH}/\omega^H) + \gamma(p^L U(c^{LH}, y^{LH}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \\ &\geq U(c^{LH}, y^{LH}/\omega^H) + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) \end{aligned} \quad (2)$$

$$\begin{aligned} &U(c^{HL}, y^{HL}/\omega^H) + \gamma(p^L U(c^{LH}, y^{LH}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \\ &\geq U(c^{LL}, y^{LL}/\omega^H) + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)). \end{aligned} \quad (3)$$

Denoting  $\mu$ ,  $\lambda_1$  and  $\lambda_2$  the Lagrange multipliers associated to the resource and incentive constraints respectively, the first-order conditions with respect to consumption and income are respectively:

$$\frac{\partial \mathcal{L}}{\partial c^{LL}} = (N^L p^L - \lambda_1 \gamma p^L - \lambda_2 - \lambda_2 \gamma p^L) u'(c_{opt}^{LL}) - \mu N^L p^L = 0 \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial c^{LH}} = (N^H p^L + \lambda_1 \gamma p^L - \lambda_1 + \lambda_2 \gamma p^L) u'(c_{opt}^{LH}) - \mu N^H p^L = 0 \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial c^{HL}} = (N^L p^H - \lambda_1 \gamma p^H + \lambda_2 - \lambda_2 \gamma p^H) u'(c_{opt}^{HL}) - \mu N^L p^H = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial c^{HH}} = (N^H p^H + \lambda_1 + \lambda_1 \gamma p^H + \lambda_2 \gamma p^H) u'(c_{opt}^{HH}) - \mu N^H p^H = 0 \quad (7)$$

and

$$\frac{\partial \mathcal{L}}{\partial y^{LL}} = (N^L p^L - \lambda_1 \gamma p^L - \lambda_2 \gamma p^L) \left(-\frac{1}{\omega^L} v'\left(\frac{y_{opt}^{LL}}{\omega^L}\right)\right) - \lambda_2 \left(-\frac{1}{\omega^H} v'\left(\frac{y_{opt}^{LL}}{\omega^H}\right)\right) + \mu N^L p^L = 0 \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial y^{LH}} = (N^H p^L + \lambda_1 \gamma p^L + \lambda_2 \gamma p^L) \left(-\frac{1}{\omega^L} v'\left(\frac{y_{opt}^{LH}}{\omega^L}\right)\right) - \lambda_1 \left(-\frac{1}{\omega^H} v'\left(\frac{y_{opt}^{LH}}{\omega^H}\right)\right) + \mu N^H p^L = 0 \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial y^{HL}} = (N^L p^H - \lambda_1 \gamma p^H + \lambda_2 - \lambda_2 \gamma p^H) \left(-\frac{1}{\omega^H} v'\left(\frac{y_{opt}^{HL}}{\omega^H}\right)\right) + \mu N^L p^H = 0 \quad (10)$$

$$\frac{\partial \mathcal{L}}{\partial y^{HH}} = (N^H p^H + \lambda_1 + \lambda_1 \gamma p^H + \lambda_2 \gamma p^H) \left(-\frac{1}{\omega^H} v'\left(\frac{y_{opt}^{HH}}{\omega^H}\right)\right) + \mu N^H p^H = 0. \quad (11)$$

The last two conditions, combined with (6) and (7), imply that the marginal utility of consumption of the  $H$  types should be equal to their marginal disutility of work. In other words, there should be no-distortion-at-the-top, a standard property in the optimal taxation literature (Stiglitz (1987)).

We describe in the next proposition some properties of the second-best allocation.

**Proposition 1.** *Second-best allocation*

1.  $c_{opt}^{HL} > c_{opt}^{LH} \forall \gamma \in [0, 1)$ ;  $c_{opt}^{HL} = c_{opt}^{LH}$  when  $\gamma \rightarrow 1$ .
2.  $c_{opt}^{HH} \geq c_{opt}^{HL} > c_{opt}^{LH}$ ,  $y_{opt}^{HL} \geq y_{opt}^{HH}$ ,  $y_{opt}^{LL} \geq y_{opt}^{LH}$ ,  $\forall \gamma \in (0, 1)$ ; When  $\gamma = 0$ ,  $c_{opt}^{HH} = c_{opt}^{HL} > c_{opt}^{LH} = c_{opt}^{LL}$ ,  $y_{opt}^{HL} = y_{opt}^{HH} > y_{opt}^{LL} = y_{opt}^{LH}$ .
3. *Incentive constraints from the high to the low types, (2) and (3), are binding at the second-best optimum, whereas incentive constraints from the low to the high types are not.*

*Proof.* See appendix. □

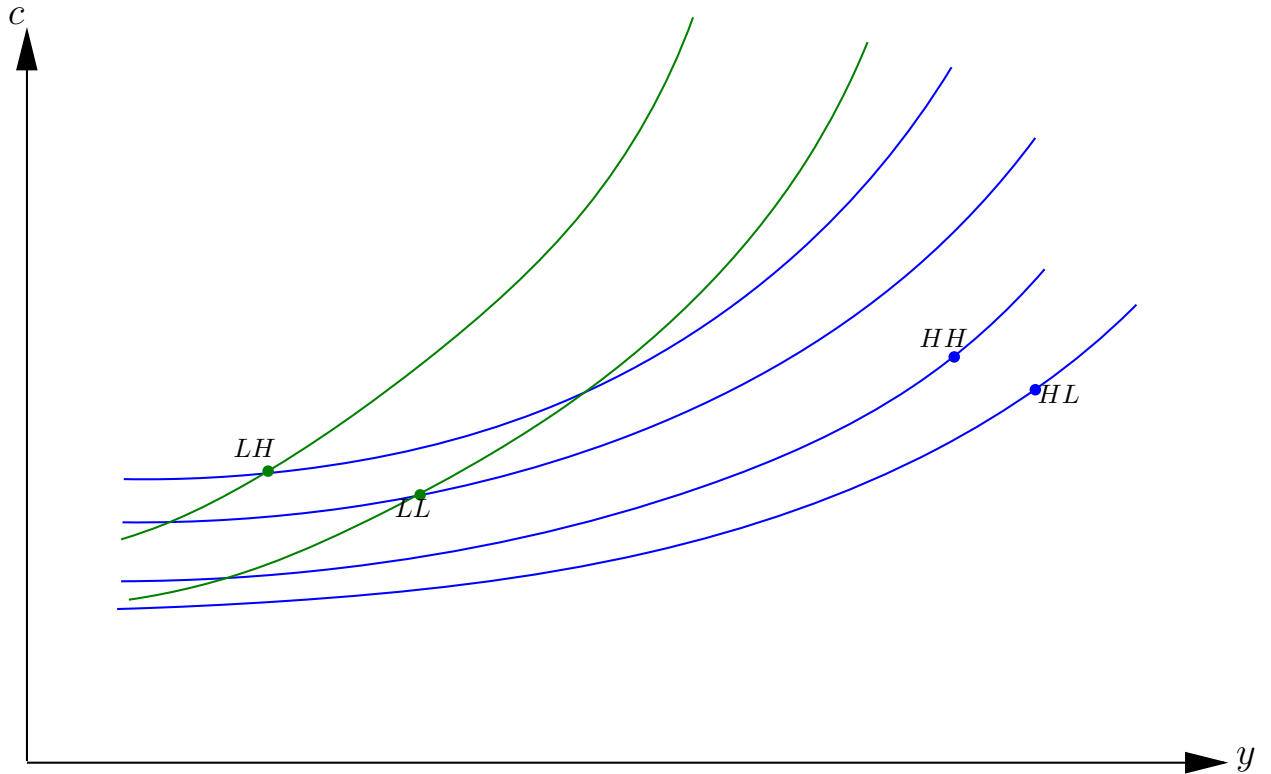


Figure 1: Second-best allocation

This proposition allows to rank the second-best allocations, as represented on figure 1. Children of highly productive individuals obtain a higher utility level than the children of individuals with a low productivity. This is explained by the incentive constraints (2) and (3): in order to provide better incentives to the  $H$  types not to mimic  $L$  types, the planner promises a higher utility level to their children.

#### 4 The optimal redistribution of bequests with independent tax schedules

We now ask which allocations can be implemented through tax schedules. The observable variables being the bequests and labor incomes, we consider taxes that depend on these two variables. With a *joint* tax schedule, that is if we allow the tax on labor income to depend on the level of bequests (and vice versa), it can be shown easily that the second-best optimum can be implemented. There is moreover some indeterminacy in the absolute level of the implementing



taxes (Farhi and Werning (2010)). We rule out this possibility and consider *separable* (nonlinear) tax functions on labor income and bequests. This means that the government is precluded from using the available information about inheritances received when it determines the tax schedule on labor income. Conversely, it cannot make the tax on bequests dependent on the level of labor income. Under this assumption, we ask whether the second-best can be implemented. Moreover, we are interested in determining if bequests should be taxed or subsidized, a long-standing controversy in the literature.

#### 4.1 Government's program

We first argue that, in a steady state with allocations depending only on the individual's type and the one of his parent, there can be only two levels of bequests: the high productivity individuals should bequeath  $b^H$  and the low productivity ones  $b^L$ . There are indeed four possible levels of consumptions and labor supplies:  $(c^{LL}, y^{LL})$ ,  $(c^{LH}, y^{LH})$ ,  $(c^{HL}, y^{HL})$  and  $(c^{HH}, y^{HH})$ . Then consider two individuals  $HH$  and  $HL$  and suppose that they respectively leave bequests  $b^{HH}$  and  $b^{HL}$ . These individuals have the same continuation value,  $p^L V^{LH} + p^H V^{HH}$ , whether they choose  $b^{HH}$  or  $b^{HL}$ . It then cannot be the case that  $b^{HH} \neq b^{HL}$ , as one of the two individuals (the one with the higher bequest) should select the bequest level intended for the other individual. The same reasoning can be made for type  $L$  individuals. Taking this into account, the government's policy  $(y^{kj}, T_y^{kj}, T_b^j, b^k)$  solves the following program:

$$\begin{aligned} \max & N^L p^L U(y^{LL} - T_y^{LL} - T_b^L, y^{LL}/\omega^L) + N^H p^L U(y^{LH} - T_y^{LH} + b^H - T_b^H - b^L, y^{LH}/\omega^L) \\ & + N^L p^H U(y^{HL} - T_y^{HL} + b^L - T_b^L - b^H, y^{HL}/\omega^H) + N^H p^H U(y^{HH} - T_y^{HH} - T_b^H, y^{HH}/\omega^H) \end{aligned} \quad (12)$$

st

$$\begin{aligned} N^L p^L T_y^{LL} + N^H p^L T_y^{LH} + N^L p^H T_y^{HL} + N^H p^H T_y^{HH} &\geq 0 \\ N^L T_b^L + N^H T_b^H &\geq 0 \end{aligned} \quad (13)$$

and

$$\begin{aligned}
& U(y^{kj} - T_y^{kj} + b^j - T_b^j - b^k, y^{kj}/\omega^k) \\
& + \gamma(p^H U(y^{Hk} - T_y^{Hk} + b^k - T_b^k - b^H, y^{Hk}/\omega^H) + p^L U(y^{Lk} - T_y^{Lk} + b^k - T_b^k - b^L, y^{Lk}/\omega^L)) \\
& \geq U(y^{k'j'} - T_y^{k'j'} + b^j - T_b^j - b^{k'}, y^{k'j'}/\omega^k) \\
& + \gamma(p^H U(y^{Hk'} - T_y^{Hk'} + b^{k'} - T_b^{k'} - b^H, y^{Hk'}/\omega^H) + p^L U(y^{Lk'} - T_y^{Lk'} + b^{k'} - T_b^{k'} - b^L, y^{Lk'}/\omega^L))
\end{aligned} \tag{14}$$

$$U(y^{kj} - T_y^{kj} + b^j - T_b^j - b^k, y^{kj}/\omega^k) \geq U(y^{k'j'} - T_y^{k'j'} + b^j - T_b^j - b^k, y^{k'j'}/\omega^k) \tag{15}$$

$$\begin{aligned}
& U(y^{kj} - T_y^{kj} + b^j - T_b^j - b^k, y^{kj}/\omega^k) \\
& + \gamma(p^H U(y^{Hk} - T_y^{Hk} + b^k - T_b^k - b^H, y^{Hk}/\omega^H) + p^L U(y^{Lk} - T_y^{Lk} + b^k - T_b^k - b^L, y^{Lk}/\omega^L)) \\
& \geq U(y^{kj} - T_y^{kj} + b^j - T_b^j - b^{k'}, y^{kj}/\omega^k) \\
& + \gamma(p^H U(y^{Hk'} - T_y^{Hk'} + b^{k'} - T_b^{k'} - b^H, y^{Hk'}/\omega^H) + p^L U(y^{Lk'} - T_y^{Lk'} + b^{k'} - T_b^{k'} - b^L, y^{Lk'}/\omega^L))
\end{aligned} \tag{16}$$

The first two constraints are the balanced-budget conditions on the tax schedules. The second group of constraints represents incentive constraints. They can be split into three sub-groups. The first constraints prevent individuals from making a *joint* deviation: type  $k$  individuals should select the pre- and post- tax labor incomes intended for them as well as the appropriate level of bequests ( $b^k$ ). The other constraints are meant to prevent *unilateral* deviations. Constraints in the second sub-group impose that individuals should not select the income tax schedule intended for other individuals while constraints in the last sub-group require that individuals select the “right” level of bequests.<sup>2</sup>

Observe that in the government’s program, only the difference  $b^H - b^L$  matters. This is denoted  $\Delta b$  in the remainder of the text. Defining  $\Omega^k$  as follows:

$$\Omega^k \equiv p^L U(c^{Lk}, y^{Lk}/\omega^L) + p^H U(c^{Hk}, y^{Hk}/\omega^H),$$

---

<sup>2</sup>Recall that, while bequests are observable, the government cannot use this information when designing the income tax schedule. Therefore an individual who has received high inheritances for example could well select the income tax schedule intended for low inheritances individuals. If we allowed the government to use all the relevant information at its disposal, this latter would propose bundles  $(y^{kj}, T_y^{kj}, T_b^j, b^k)$  and  $(y^{k'j'}, T_y^{k'j'}, T_b^j, b^{k'})$  to an individual who has received inheritances  $b^j$ . The program of the government would then be identical to the second-best problem, implying that the optimal tax implements the second-best allocation.

we can thus write the constraints preventing joint deviations as follows:

$$u(c^{HH}) - v\left(\frac{y^{HH}}{\omega^H}\right) + \gamma\Omega^H \geq u(c^{LH}) - v\left(\frac{y^{LH}}{\omega^H}\right) + \gamma\Omega^L \quad (17)$$

$$u(c^{HH}) - v\left(\frac{y^{HH}}{\omega^H}\right) \geq u(c^{HL} + T_b^L + \Delta b - T_b^H) - v\left(\frac{y^{HL}}{\omega^H}\right) \quad (18)$$

$$u(c^{HH}) - v\left(\frac{y^{HH}}{\omega^H}\right) + \gamma\Omega^H \geq u(c^{LL} + T_b^L + \Delta b - T_b^H) - v\left(\frac{y^{LL}}{\omega^H}\right) + \gamma\Omega^L \quad (19)$$

$$u(c^{HL}) - v\left(\frac{y^{HL}}{\omega^H}\right) + \gamma\Omega^H \geq u(c^{LL}) - v\left(\frac{y^{LL}}{\omega^H}\right) + \gamma\Omega^L \quad (20)$$

$$u(c^{HL}) - v\left(\frac{y^{HL}}{\omega^H}\right) \geq u(c^{HH} - T_b^L - \Delta b + T_b^H) - v\left(\frac{y^{HH}}{\omega^H}\right) \quad (21)$$

$$u(c^{HL}) - v\left(\frac{y^{HL}}{\omega^H}\right) + \gamma\Omega^H \geq u(c^{LH} - T_b^L - \Delta b + T_b^H) - v\left(\frac{y^{LH}}{\omega^H}\right) + \gamma\Omega^L \quad (22)$$

$$u(c^{LH}) - v\left(\frac{y^{LH}}{\omega^L}\right) + \gamma\Omega^L \geq u(c^{HH}) - v\left(\frac{y^{HH}}{\omega^L}\right) + \gamma\Omega^H \quad (23)$$

$$u(c^{LH}) - v\left(\frac{y^{LH}}{\omega^L}\right) + \gamma\Omega^L \geq u(c^{HL} + T_b^L + \Delta b - T_b^H) - v\left(\frac{y^{HL}}{\omega^L}\right) + \gamma\Omega^H \quad (24)$$

$$u(c^{LH}) - v\left(\frac{y^{LH}}{\omega^L}\right) \geq u(c^{LL} + T_b^L + \Delta b - T_b^H) - v\left(\frac{y^{LL}}{\omega^L}\right) \quad (25)$$

$$u(c^{LL}) - v\left(\frac{y^{LL}}{\omega^L}\right) + \gamma\Omega^L \geq u(c^{HL}) - v\left(\frac{y^{HL}}{\omega^L}\right) + \gamma\Omega^H \quad (26)$$

$$u(c^{LL}) - v\left(\frac{y^{LL}}{\omega^L}\right) + \gamma\Omega^L \geq u(c^{HH} - T_b^L - \Delta b + T_b^H) - v\left(\frac{y^{HH}}{\omega^L}\right) + \gamma\Omega^H \quad (27)$$

$$u(c^{LL}) - v\left(\frac{y^{LL}}{\omega^L}\right) \geq u(c^{LH} - T_b^L - \Delta b + T_b^H) - v\left(\frac{y^{LH}}{\omega^L}\right). \quad (28)$$

The constraints on the income tax schedule are the following:<sup>3</sup>

$$u(c^{HH}) - v\left(\frac{y^{HH}}{\omega^H}\right) \geq u(c^{LH} - \Delta b) - v\left(\frac{y^{LH}}{\omega^H}\right) \quad (29)$$

$$u(c^{HH}) - v\left(\frac{y^{HH}}{\omega^H}\right) \geq u(c^{LL} + T_b^L - T_b^H) - v\left(\frac{y^{LL}}{\omega^H}\right) \quad (30)$$

$$u(c^{HL}) - v\left(\frac{y^{HL}}{\omega^H}\right) \geq u(c^{LL} - \Delta b) - v\left(\frac{y^{LL}}{\omega^H}\right) \quad (31)$$

$$u(c^{HL}) - v\left(\frac{y^{HL}}{\omega^H}\right) \geq u(c^{LH} - T_b^L - 2\Delta b + T_b^H) - v\left(\frac{y^{LH}}{\omega^H}\right) \quad (32)$$

$$u(c^{LH}) - v\left(\frac{y^{LH}}{\omega^L}\right) \geq u(c^{HH} + \Delta b) - v\left(\frac{y^{HH}}{\omega^L}\right) \quad (33)$$

$$u(c^{LH}) - v\left(\frac{y^{LH}}{\omega^L}\right) \geq u(c^{HL} + T_b^L + 2\Delta b - T_b^H) - v\left(\frac{y^{HL}}{\omega^L}\right) \quad (34)$$

$$u(c^{LL}) - v\left(\frac{y^{LL}}{\omega^L}\right) \geq u(c^{HL} + \Delta b) - v\left(\frac{y^{HL}}{\omega^L}\right) \quad (35)$$

$$u(c^{LL}) - v\left(\frac{y^{LL}}{\omega^L}\right) \geq u(c^{HH} - T_b^L + T_b^H) - v\left(\frac{y^{HH}}{\omega^L}\right) \quad (36)$$

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<sup>3</sup>Some of the constraints, which are redundant with the joint deviations constraints, have been omitted.

Finally, the constraints ensuring that individuals select the appropriate level of bequests are:

$$U(c^{HH}, y^{HH}/\omega^H) + \gamma\Omega^H \geq U(c^{HH} + \Delta b, y^{HH}/\omega^H) + \gamma\Omega^L \quad (37)$$

$$U(c^{HL}, y^{HL}/\omega^H) + \gamma\Omega^H \geq U(c^{HL} + \Delta b, y^{HL}/\omega^H) + \gamma\Omega^L \quad (38)$$

$$U(c^{LH}, y^{LH}/\omega^L) + \gamma\Omega^L \geq U(c^{LH} - \Delta b, y^{LH}/\omega^L) + \gamma\Omega^H \quad (39)$$

$$U(c^{LL}, y^{LL}/\omega^L) + \gamma\Omega^L \geq U(c^{LL} - \Delta b, y^{LL}/\omega^L) + \gamma\Omega^H. \quad (40)$$

Writing the objective function (12) and the income tax budget constraint (13) in terms of consumption and income levels, the government program can be solved by choosing optimally  $(y^{kj}, c^{kj}, T_b^j, b^k)$ .

## 4.2 Implementability of the second best

We show in the following proposition, which proof is in the appendix, that the second-best allocation cannot be implemented with separable tax schedules on labor income and bequests.

**Proposition 2.** *The second-best allocation cannot be implemented with separable tax schedules on bequests and labor income.*

The proof consists in showing that constraint (19) is always binding at the second-best allocation.

## 4.3 Optimal taxes on bequests

We now determine the optimal taxes on bequests. This is done by solving the government program in two steps. We first determine the optimal policy - income, consumption and bequests levels - when there is no taxation of bequests ( $T_b^L = T_b^H = 0$ ). We then introduce (redistributive) taxes on bequests. This allows to conclude about the desirability of taxing bequests and in which direction the redistribution of bequests should take place.

The impact of introducing a small tax on high bequests, the proceeds of which are redis-

tributed to low bequests individuals, is given by:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial T_b^H} \Big|_{T_b^L=T_b^H=0} &= \left(1 + \frac{N^H}{N^L}\right) \left( \right. \\ & u'(c^{LL} + b^H)(\lambda^{HHLL} + \lambda^{LHLL} + \beta^{LHLL}) + \beta^{HHLL} u'(c^{LL}) \\ & - u'(c^{LH} - b^H)(\lambda^{HLLH} + \lambda^{LLLH} + \beta^{LLLH}) - \beta^{HLLH} u'(c^{LH} - 2b^H) \\ & + u'(c^{HL} + b^H)(\lambda^{HHHL} + \lambda^{LHHL} + \beta^{HHHL}) + \beta^{LHHL} u'(c^{HL} + 2b^H) \\ & \left. - u'(c^{HH} - b^H)(\lambda^{HLHH} + \lambda^{LLHH} + \beta^{HLHH}) - \beta^{LLHH} u'(c^{HH}) \right), \end{aligned}$$

where  $\lambda$  and  $\beta$  are the Lagrange multipliers associated with the joint and labor income incentive constraints respectively. Substituting the first-order condition on bequests:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial b^H} \Big|_{T_b^L=T_b^H=0} &= -u'(c^{LL} + b^H)(\lambda^{HHLL} + \lambda^{LHLL} + \beta^{LHLL}) \\ & + \beta^{HLLL} u'(c^{LL} - b^H) + \delta^{LLH} u'(c^{LL} - b^H) \\ & + u'(c^{LH} - b^H)(\lambda^{HLLH} + \lambda^{LLLH} + \beta^{LLLH}) \\ & + 2\beta^{HLLH} u'(c^{LH} - 2b^H) + \beta^{HHLH} u'(c^{LH} - b^H) + \delta^{LHH} u'(c^{LH} - b^H) \\ & - u'(c^{HL} + b^H)(\lambda^{HHHL} + \lambda^{LHHL} + \beta^{HHHL}) \\ & - 2\beta^{LHHL} u'(c^{HL} + 2b^H) - \beta^{LLHL} u'(c^{HL} + b^H) - \delta^{HLL} u'(c^{HL} + b^H) \\ & + u'(c^{HH} - b^H)(\lambda^{HLHH} + \lambda^{LLHH} + \beta^{HLHH}) \\ & - \beta^{LHHH} u'(c^{HH} + b^H) - \delta^{HHL} u'(c^{HH} + b^H) = 0, \end{aligned}$$

this can be rewritten as:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial T_b^H} \Big|_{T_b^L=T_b^H=0} &= \left(1 + \frac{N^H}{N^L}\right) \left( \right. \\ & \beta^{HLLL} u'(c^{LL} - b^H) + \delta^{LLH} u'(c^{LL} - b^H) + \beta^{HHLL} u'(c^{LL}) \\ & + \beta^{HLLH} u'(c^{LH} - 2b^H) + \beta^{HHLH} u'(c^{LH} - b^H) + \delta^{LHH} u'(c^{LH} - b^H) \\ & - \beta^{LHHL} u'(c^{HL} + 2b^H) - \beta^{LLHL} u'(c^{HL} + b^H) - \delta^{HLL} u'(c^{HL} + b^H) \\ & \left. - \beta^{LHHH} u'(c^{HH} + b^H) - \delta^{HHL} u'(c^{HH} + b^H) - \beta^{LLHH} u'(c^{HH}) \right), \quad (41) \end{aligned}$$

where  $\delta$  denotes the Lagrange multiplier associated with the bequests incentive constraints. It appears from this condition that the joint deviation constraints play no role in determining the sign of the optimal tax on bequests. This sign depends on the constraints ensuring that the individuals choose the appropriate level of bequests as well as a subset of the constraints on

labor income. In the following lemma, we prove that some of these constraints can never be binding and provide conditions for the other constraints to be binding.

**Lemma 1.** *At the tax optimum with  $T_b^L = T_b^H = 0$ :*

1. *The constraints (29), (31), (32), (33), (34), (35), (39) and (40) cannot be binding.*
2. *When (30) binds, constraints (36), (37) and (38) cannot be binding.*

*Proof.* See appendix. □

Using the first part of lemma 1, we can rewrite (41) as:

$$\left. \frac{\partial \mathcal{L}}{\partial T_b^H} \right|_{T_b^L=T_b^H=0} = \left(1 + \frac{N^H}{N^L}\right) (\beta^{HHLL} u'(c^{LL}) - \delta^{HLL} u'(c^{HL} + b^H) - \delta^{HHL} u'(c^{HH} + b^H) - \beta^{LLHH} u'(c^{HH})).$$

Moreover, the second part of lemma 1 implies that  $\beta^{LLHH} = \delta^{HHL} = \delta^{HLL} = 0$  when  $\beta^{HHLL} > 0$ . In such a case, we have:

$$\left. \frac{\partial \mathcal{L}}{\partial T_b^H} \right|_{T_b^L=T_b^H=0} = \left(1 + \frac{N^H}{N^L}\right) \beta^{HHLL} u'(c^{LL}) > 0.$$

In words, *high bequests should be taxed* when (30) is binding, the proceeds of this tax being redistributed to individuals having received low bequests. On the other hand, when (31) does not bind ( $\beta^{HHLL} = 0$ ), it may be possible that  $\beta^{LLHH}$ ,  $\delta^{HLL}$  or  $\delta^{HHL}$  are positive. In such a case, either no taxation of bequests is desirable or *high bequests should be subsidized*. These results are summarized in the following proposition.

**Proposition 3.** *High bequests should be taxed (and low bequests subsidized) if and only if (30) binds at the optimum with taxes when  $T_b^L$  and  $T_b^H$  are set to 0. When it does not bind, either no taxation of bequests is desirable or high bequests should be subsidized.*

This proposition puts forward the crucial role played by constraint (30). We argue in the next proposition that constraint (30) binds, and therefore that large bequests should be taxed, as soon as the degree of altruism,  $\gamma$ , and the proportion of low productivity individuals,  $p^L$ , are large enough.

**Proposition 4.** *When  $\gamma \rightarrow 1$  and  $p^L \rightarrow 1$ , it is optimal to tax the bequests of the high productivity individuals and to subsidize the bequests of the low productivity individuals.*

$\gamma$	Binding constraints when $T_b^L = T_b^H = 0$	Binding constraints with opt. taxes
1	ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_hl_ll, ic_joint_lh_ll ic_inc_hh_ll, ic_inc_hl_hh, ic_inc_lh_ll	ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_hl_ll, ic_joint_ll_lh ic_inc_hl_hh, ic_inc_ll_lh
0.75	ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_ll_lh ic_inc_hl_hh, ic_inc_ll_lh	ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_ll_lh ic_inc_hl_hh, ic_inc_ll_lh
0.5	ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_ll_lh ic_inc_hl_hh, ic_inc_ll_lh ic_beq_hh_l, ic_beq_hl_l	ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_ll_lh ic_inc_hl_hh, ic_inc_ll_lh ic_beq_hh_l, ic_beq_hl_l
0	ic_joint_hh_hl, ic_joint_hh_lh, ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_hl_lh, ic_joint_hl_ll, ic_joint_lh_ll, ic_joint_ll_lh ic_inc_hh_hl, ic_inc_hh_lh, ic_inc_hh_ll, ic_inc_hl_hh, ic_inc_hl_lh, ic_inc_hl_ll, ic_inc_lh_ll, ic_inc_ll_lh ic_beq_hh_l, ic_beq_hl_l, ic_beq_lh_h, ic_beq_ll_h	ic_joint_hh_hl, ic_joint_hh_lh, ic_joint_hh_ll, ic_joint_hl_hh, ic_joint_hl_lh, ic_joint_hl_ll, ic_joint_lh_ll, ic_joint_ll_lh ic_inc_hh_hl, ic_inc_hh_lh, ic_inc_hh_ll, ic_inc_hl_hh, ic_inc_hl_lh, ic_inc_hl_ll, ic_inc_lh_ll, ic_inc_ll_lh ic_beq_hh_l, ic_beq_hl_l, ic_beq_lh_h, ic_beq_ll_h

Table 1: Binding constraints

*Proof.* See appendix. □

When the altruism parameter or the proportion of type  $L$  individuals take moderate or low values, it may be possible that the constraint (30) does not bind. In such a case, the optimal tax on the bequests of the type  $H$  individuals may be 0 or even negative. We present in the next section a numerical example that confirms this possibility.

## 5 Numerical illustration

We adopt the same utility function as Mankiw et al. (2009):

$$U(c, l) = \frac{c^{1-\beta} - 1}{1-\beta} - \frac{\alpha l^\sigma}{\sigma},$$

with  $\beta = 1.5$ ,  $\alpha = 2.55$  and  $\sigma = 3$ .

Productivity levels are  $\omega^L = 100$  and  $\omega^H = 200$ . The low productivity individuals are twice as numerous as high productivity individuals:  $N^L = 20$  and  $N^H = 10$ . In the absence of intergenerational correlation, this implies  $p^L = 2/3 > p^H = 1/3$ . Finally the altruism parameter,  $\gamma$ , takes four possible values: 0, 0.5, 0.75 and 1.

The results presented in table 2 confirm the findings of proposition 4: for a large enough value of  $\gamma$  ( $\gamma \rightarrow 1$ ), the bequests of the high productivity individuals should be taxed. For lower values of  $\gamma$  ( $\gamma = 0.5$  and  $\gamma = 0.75$ ), the constraint (30) does not bind. In such a case, either no

$\gamma$	Optimal taxes and bequests		Allocation with opt. taxes		Second best allocation	
1	$b^L = 4.740$	$T_b^L = -0.438$	$y^{LL} = 34.795$	$y^{LH} = 30.287$	$y^{LL} = 34.674$	$y^{LH} = 30.354$
	$b^H = 21.514$	$T_b^H = +0.876$	$y^{HL} = 89.590$	$y^{HH} = 77.428$	$y^{HL} = 88.163$	$y^{HH} = 80.926$
	$T_y^{LL} = -9.028$	$T_y^{LH} = -10.153$	$c^{LL} = 44.261$	$c^{LH} = 56.339$	$c^{LL} = 44.242$	$c^{LH} = 54.616$
	$T_y^{HL} = +19.794$	$T_y^{HH} = +21.514$	$c^{HL} = 53.461$	$c^{HH} = 59.721$	$c^{HL} = 54.616$	$c^{HH} = 61.223$
			$U^{LL} = 1.664$	$U^{LH} = 1.710$	$U^{LL} = 1.664$	$U^{LH} = 1.706$
		$U^{HL} = 1.650$	$U^{HH} = 1.692$	$U^{HL} = 1.657$	$U^{HH} = 1.688$	
0.75	$b^L = 3.445$	$T_b^L = 0$	$y^{LL} = 35.109$	$y^{LH} = 30.547$	$y^{LL} = 34.833$	$y^{LH} = 30.619$
	$b^H = 19.227$	$T_b^H = 0$	$y^{HL} = 88.029$	$y^{HH} = 74.386$	$y^{HL} = 86.691$	$y^{HH} = 79.488$
	$T_y^{LL} = -7.740$	$T_y^{LH} = -8.986$	$c^{LL} = 42.849$	$c^{LH} = 55.315$	$c^{LL} = 43.247$	$c^{LH} = 53.020$
	$T_y^{HL} = +17.520$	$T_y^{HH} = +13.891$	$c^{HL} = 54.727$	$c^{HH} = 60.495$	$c^{HL} = 55.855$	$c^{HH} = 62.704$
			$U^{LL} = 1.658$	$U^{LH} = 1.707$	$U^{LL} = 1.660$	$U^{LH} = 1.701$
		$U^{HL} = 1.657$	$U^{HH} = 1.699$	$U^{HL} = 1.663$	$U^{HH} = 1.694$	
0.5	$b^L = 5.670$	$T_b^L = +0.787$	$y^{LL} = 34.468$	$y^{LH} = 32.690$	$y^{LL} = 34.843$	$y^{LH} = 31.208$
	$b^H = 12.643$	$T_b^H = -1.574$	$y^{HL} = 83.085$	$y^{HH} = 74.794$	$y^{HL} = 84.699$	$y^{HH} = 78.348$
	$T_y^{LL} = -7.723$	$T_y^{LH} = -8.172$	$c^{LL} = 41.396$	$c^{LH} = 49.401$	$c^{LL} = 42.385$	$c^{LH} = 50.420$
	$T_y^{HL} = +16.231$	$T_y^{HH} = +14.774$	$c^{HL} = 59.111$	$c^{HH} = 61.611$	$c^{HL} = 57.614$	$c^{HH} = 63.924$
			$U^{LL} = 1.654$	$U^{LH} = 1.686$	$U^{LL} = 1.657$	$U^{LH} = 1.693$
		$U^{HL} = 1.679$	$U^{HH} = 1.701$	$U^{HL} = 1.672$	$U^{HH} = 1.699$	
0	$b^L = 9.808$	$T_b^L = 0$	$y^{LL} = 33.863$	$y^{LH} = 33.863$	$y^{LL} = 33.863$	$y^{LH} = 33.863$
	$b^H = 9.808$	$T_b^H = 0$	$y^{HL} = 80.239$	$y^{HH} = 80.239$	$y^{HL} = 80.239$	$y^{HH} = 80.239$
	$T_y^{LL} = -9.158$	$T_y^{LH} = -9.158$	$c^{LL} = 43.021$	$c^{LH} = 43.021$	$c^{LL} = 43.021$	$c^{LH} = 43.021$
	$T_y^{HL} = +18.317$	$T_y^{HH} = +18.317$	$c^{HL} = 61.923$	$c^{HH} = 61.923$	$c^{HL} = 61.923$	$c^{HH} = 61.923$
			$U^{LL} = 1.662$	$U^{LH} = 1.662$	$U^{LL} = 1.662$	$U^{LH} = 1.662$
		$U^{HL} = 1.691$	$U^{HH} = 1.691$	$U^{HL} = 1.691$	$U^{HH} = 1.691$	

Table 2: Optimal taxes and allocations



taxation of bequests is desirable or large bequests should be subsidized (Proposition 3). This latter case occurs when  $\gamma = 0.5$ , the reason being that constraints (37) and (38) are binding. In contrast, these constraints are not binding when  $\gamma = 0.75$ , implying that bequests should not be taxed.

## 6 Conclusion

Our analysis has contributed to shed light on the optimal level of bequests taxation. We have shown that, depending on the level of altruism, high bequests should be taxed or subsidized. The former case occurs when individuals are altruistic enough and low productivity individuals are sufficiently numerous. For intermediate and low values of the altruism parameter, either no taxation of bequests is desirable or large bequests should be subsidized.

These results were obtained in a stylized framework and more work is needed to fully characterize the optimal tax schedule on bequests. In our view, four main avenues of research should be envisaged. First, we have developed a model with two productivity levels. A natural extension would consist in determining the optimal tax schedule when productivities are continuously distributed, as in the standard Mirrlees framework (Mirrlees (1971)). This would allow for a careful examination of the way marginal tax rates vary with the bequest level. Second, it was assumed that productivities were not correlated between parents and children. We would like to relax this assumption in future research, in order to gain a better understanding of how the optimal tax on bequests varies with the degree of intergenerational correlation. Third, the results were obtained in the specific case where the steady state allocation received by a given individual is constrained to depend on his type and the type of his parent only. The dependence of allocations on longer histories should be dealt with in future work. Finally, a single heterogeneity between individuals, on the productivity levels, was taken into account. We believe that other dimensions of heterogeneity may play an important role in such an intergenerational context, notably differences with respect to altruism between individuals, and the way it correlates with productivity.

# Appendix

## A Program of the social planner

The utilitarian objective in the steady state is:

$$N^L p^L V^{LL} + N^H p^L V^{LH} + N^L p^H V^{HL} + N^H p^H V^{HH}$$

where

$$V^{ij} = U^{ij} + \gamma \sum_k p^k V^{ki}.$$

It can thus be rewritten as:

$$\begin{aligned} & N^L p^L U^{LL} + N^H p^L U^{LH} + N^L p^H U^{HL} + N^H p^H U^{HH} \\ + & N^L p^L \gamma (p^L V^{LL} + p^H V^{HL}) \\ + & N^H p^L \gamma (p^L V^{LL} + p^H V^{HL}) \\ + & N^L p^H \gamma (p^L V^{LH} + p^H V^{HH}) \\ + & N^H p^H \gamma (p^L V^{LH} + p^H V^{HH}) \\ = & N^L p^L U^{LL} + N^H p^L U^{LH} + N^L p^H U^{HL} + N^H p^H U^{HH} \\ + & N p^L \gamma (p^L V^{LL} + p^H V^{HL}) + N p^H \gamma (p^L V^{LH} + p^H V^{HH}) \\ = & N^L p^L U^{LL} + N^H p^L U^{LH} + N^L p^H U^{HL} + N^H p^H U^{HH} \\ + & \gamma (N^L p^L V^{LL} + N^H p^L V^{LH} + N^L p^H V^{HL} + N^H p^H V^{HH}) \\ = & N^L p^L U^{LL} + N^H p^L U^{LH} + N^L p^H U^{HL} + N^H p^H U^{HH} \\ + & \gamma (N^L p^L U^{LL} + N^H p^L U^{LH} + N^L p^H U^{HL} + N^H p^H U^{HH}) \\ + & \gamma^2 (N^L p^L V^{LL} + N^H p^L V^{LH} + N^L p^H V^{HL} + N^H p^H V^{HH}) \\ = & (1 + \gamma + \gamma^2 + \dots) (N^L p^L U^{LL} + N^H p^L U^{LH} + N^L p^H U^{HL} + N^H p^H U^{HH}) \\ = & (1/(1 - \gamma)) (N^L p^L U^{LL} + N^H p^L U^{LH} + N^L p^H U^{HL} + N^H p^H U^{HH}). \end{aligned}$$

Hence the objective function in the main text, where the term  $1/(1 - \gamma)$  has been removed.

## B Proof of proposition 1

1. At the second-best allocation, at least one of the two incentive constraints (2) and (3) is necessarily binding, otherwise the first-best could be implemented. We prove that in fact both constraints are. Suppose that (3) is not ( $\lambda_2 = 0$ ); (4) and (6) then write:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c^{LL}} &= (N^L p^L - \lambda_1 \gamma p^L) u'(c_{opt}^{LL}) - \mu N^L p^L = 0 \\ \frac{\partial \mathcal{L}}{\partial c^{HL}} &= (N^L p^H - \lambda_1 \gamma p^H) u'(c_{opt}^{HL}) - \mu N^L p^H = 0.\end{aligned}$$

Dividing these two conditions by  $N^L p^L$  and  $N^L p^H$  respectively, it appears that they are identical and thus that  $c^{LL} = c^{HL}$ . This, using (4), would imply:

$$\left. \frac{\partial \mathcal{L}}{\partial c^{HL}} \right|_{c^{HL}=c^{LL}} = \left( \frac{\lambda_2}{N^L p^H} + \frac{\lambda_2}{N^L p^L} \right) u'(c^{LL}).$$

Setting  $\lambda_2 > 0$  and increasing  $c^{HL}$  would therefore allow to increase social welfare. A similar reasoning can be made to show that one cannot have  $\lambda_2 \neq 0$  and  $\lambda_1 = 0$ . Knowing that (2) and (3) are binding, standard argument can be used to prove that constraints from the low to the high types cannot be binding.

2. We evaluate  $\partial \mathcal{L} / \partial c^{HL}$  at the point  $c^{HL} = c^{LH}$ :

$$\left. \frac{\partial \mathcal{L}}{\partial c^{HL}} \right|_{c^{HL}=c^{LH}} = N^L p^H u'(c^{LH}) - \mu N^L p^H - \lambda_1 \gamma p^H u'(c^{LH}) + \lambda_2 u'(c^{LH}) - \lambda_2 \gamma p^H u'(c^{LH}).$$

Noting that  $N^L p^H = N^H p^L$  and using (5), we obtain:

$$\begin{aligned}\left. \frac{\partial \mathcal{L}}{\partial c^{HL}} \right|_{c^{HL}=c^{LH}} &= -\lambda_1 \gamma p^L u'(c^{LH}) + \lambda_1 u'(c^{LH}) - \lambda_2 \gamma p^L u'(c^{LH}) \\ &\quad - \lambda_1 \gamma p^H u'(c^{LH}) + \lambda_2 u'(c^{LH}) - \lambda_2 \gamma p^H u'(c^{LH}) \\ &= -\lambda_1 \gamma u'(c^{LH}) + \lambda_1 u'(c^{LH}) - \lambda_2 \gamma u'(c^{LH}) + \lambda_2 u'(c^{LH}) \geq 0.\end{aligned}$$

Therefore  $c^{HL} > c^{LH} \forall \gamma \in [0, 1)$  and  $c^{HL} = c^{LH}$  when  $\gamma \rightarrow 1$ .

3. Suppose that  $c^{HH} < c^{HL}$ . Dividing (6) and (7) by  $N^L p^H$  and  $N^H p^H$  respectively and comparing these two expressions, one must have:

$$\frac{\lambda_1}{N^H p^H} + \frac{\lambda_1 \gamma}{N^H} + \frac{\lambda_2 \gamma}{N^H} < \frac{\lambda_2}{N^L p^H} - \frac{\lambda_1 \gamma}{N^L} - \frac{\lambda_2 \gamma}{N^L}, \quad (42)$$

Inspecting (10) and (11), this relationship implies  $y^{HH} > y^{HL}$ . This means that  $(y^{HL}, c^{HL})$  is on a higher indifference curve than  $(y^{HH}, c^{HH})$ . From the incentive constraints (2) and (3), it must then be that  $(y^{LL}, c^{LL})$  is on a higher indifference curve of the  $H$  types than  $(y^{LH}, c^{LH})$ . Inspecting (4) and (5), one can check that  $c^{LL} < c^{LH}$  when (42) holds true. For  $(y^{LL}, c^{LL})$  to be on a higher indifference curve of the  $H$  types than  $(y^{LH}, c^{LH})$ , it must then be that  $y^{LL} < y^{LH}$ . However when  $\lambda_2/N^L > \lambda_1/N^H$ , which is implied by (42), the inspection of (8) and (9) makes clear that  $y^{LL} > y^{LH}$ , a contradiction. We thus have shown that  $c^{HH} \geq c^{HL}$ . This necessarily involves:

$$\frac{\lambda_1}{N^H p^H} + \frac{\lambda_1 \gamma}{N^H} + \frac{\lambda_2 \gamma}{N^H} \geq \frac{\lambda_2}{N^L p^H} - \frac{\lambda_1 \gamma}{N^L} - \frac{\lambda_2 \gamma}{N^L}.$$

Under this condition, (10) and (11) imply  $y^{HH} \leq y^{HL}$ .

4. Suppose that  $y^{LL} < y^{LH}$ . From (8) and (9), this is possible if and only if:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial y^{LL}} \Big|_{y^{LL}=y^{LH}} &< 0 \\ \Leftrightarrow \lambda_1 N^L - \lambda_2 N^H &> \frac{\frac{1}{\omega^L} v'(\frac{y^{LL}}{\omega^L})}{\frac{1}{\omega^H} v'(\frac{y^{LL}}{\omega^H})} \gamma N^L (\lambda_1 + \lambda_2). \end{aligned} \quad (43)$$

Observe that, as a consequence of 3.,  $U(c^{HH}, y^{HH}/\omega^H) \geq U(c^{HL}, y^{HL}/\omega^H)$ . The binding incentive constraints (2) and (3) then imply  $U(c^{LH}, y^{LH}/\omega^H) \geq U(c^{LL}, y^{LL}/\omega^H)$ . When  $y^{LL} < y^{LH}$ , this is possible only if  $c^{LL} < c^{LH}$ . Combining (4) and (5), a necessary and sufficient condition for having  $c^{LL} < c^{LH}$  is:

$$\lambda_1 N^L - \lambda_2 N^H < \gamma N^L (\lambda_1 + \lambda_2).$$

Noting that  $1/\omega v'(y/\omega)$  is decreasing with  $\omega$ , this condition is not compatible with (43), meaning that one cannot have  $y^{LL} < y^{LH}$ .

5. When  $\gamma = 0$ , it can be checked that the first-order conditions for the second-best optimum are satisfied when  $(y^{LH}, c^{LH}) = (y^{LL}, c^{LL})$ ,  $(y^{HH}, c^{HH}) = (y^{HL}, c^{HL})$  and  $\lambda_1/N^H = \lambda_2/N^L$ .

## C Proof of proposition 2

We have shown previously that (2) is binding at the second-best allocation:

$$U(c^{HH}, y^{HH}/\omega^H) + \gamma \Omega^H = U(c^{LH}, y^{LH}/\omega^H) + \gamma \Omega^L.$$

It follows that (19) can be satisfied iff:

$$U(c^{LH}, y^{LH}/\omega^H) \geq U(c^{LL} + T_b^L + \Delta b - T_b^H, y^{LL}/\omega^H).$$

This condition is satisfied iff  $\Delta b \leq b_1$ , with  $b_1$  implicitly defined by:

$$U(c^{LH}, y^{LH}/\omega^H) = U(c^{LL} + T_b^L + b_1 - T_b^H, y^{LL}/\omega^H) \quad (44)$$

We then argue that this condition is not compatible with (28). This latter constraint indeed imposes that  $\Delta b \geq b_2$ , where  $b_2$  is implicitly defined by:

$$U(c^{LL}, y^{LL}/\omega^L) = U(c^{LH} - T_b^L - b_2 + T_b^H, y^{LH}/\omega^L).$$

A graphical inspection makes clear that  $b_2 > b_1$  and thus that the two conditions are not compatible. Formally this can be shown by differentiating (44):

$$\frac{db_1}{d\omega^H} = \frac{y^{LH}v'(y^{LH}/\omega^H) - y^{LL}v'(y^{LL}/\omega^H)}{(\omega^H)^2u'(c^{LL} + b^H)}.$$

This expression is, recalling that  $v$  is convex and that  $y^{LL} > y^{LH}$ , negative. Noting that  $b_1 = b_2$  when  $\omega^L = \omega^H$ , this implies that  $b_1 < b_2$ . qed.

## D Proof of lemma 1

### D.1 Ranking of allocations at the tax optimum

1.  $y^{LH} \leq y^{LL}$  at the optimum with taxes.

Define  $\tilde{c}^{LL-LH}$  and  $\tilde{c}^{LH-LL}$  as the consumption levels that satisfy respectively:

$$U(\tilde{c}^{LL-LH}, y^{LH}/\omega^L) = U(c^{LL}, y^{LL}/\omega^L)$$

and

$$U(\tilde{c}^{LH-LL}, y^{LL}/\omega^L) = U(c^{LH}, y^{LH}/\omega^L).$$

Condition (28) implies:

$$\Delta b - T_b^H + T_b^L \geq c^{LH} - \tilde{c}^{LL-LH}. \quad (45)$$

Condition (25) implies:

$$\Delta b - T_b^H + T_b^L \leq \tilde{c}^{LH-LL} - c^{LL}. \quad (46)$$

With a separable utility and  $u(\cdot)$  strictly concave, the distance between indifference curves increases with  $y$ . Therefore  $y^{LH} > y^{LL}$  would imply  $c^{LH} - \tilde{c}^{LL-LH} > \tilde{c}^{LH-LL} - c^{LL}$ , which contradicts the two inequalities above. Hence  $y^{LH} \leq y^{LL}$ .

2.  $y^{HH} \leq y^{HL}$  at the optimum with taxes.

Define  $\tilde{c}^{HH-HL}$  and  $\tilde{c}^{HL-HH}$  as the consumption levels that satisfy:

$$U(\tilde{c}^{HH-HL}, y^{HL}/\omega^H) = U(c^{HH}, y^{HH}/\omega^H)$$

and

$$U(\tilde{c}^{HL-HH}, y^{HH}/\omega^H) = U(c^{HL}, y^{HL}/\omega^H).$$

Condition (18) implies:

$$\Delta b - T_b^H + T_b^L \leq \tilde{c}^{HH-HL} - c^{HL}. \quad (47)$$

Condition (21) implies:

$$\Delta b - T_b^H + T_b^L \geq c^{HH} - \tilde{c}^{HL-HH}.$$

With a separable utility and  $u(\cdot)$  strictly concave, the distance between indifference curves increases with  $y$ . Therefore these two inequalities are compatible only if  $y^{HH} \leq y^{HL}$ .

3.  $y^{HH} \geq y^{LL}$  and  $c^{HH} \geq c^{LL}$  at the optimum with taxes when  $T_b^L = T_b^H = 0$ .

When  $T_b^L = T_b^H = 0$ , condition (30) implies that the type  $H$  indifference curves passing through  $(c^{HH}, y^{HH})$  is above the one passing through  $(c^{LL}, y^{LL})$ . Then having  $y^{LL} > y^{HH}$  would violate the incentive constraint (36). Hence  $y^{HH} \geq y^{LL}$  when  $T_b^L = T_b^H = 0$ . Condition (30), then implies  $c^{HH} \geq c^{LL}$ .

## D.2 Binding labor income constraints at the tax optimum

1. Condition (29) cannot be binding at the optimum with taxes when  $T_b^L = T_b^H = 0$ .

Define  $\tilde{c}^{HH-LH}$  and as the consumption level that satisfies:

$$U(\tilde{c}^{HH-LH}, y^{LH}/\omega^H) = U(c^{HH}, y^{HH}/\omega^H).$$

Condition (30), combined with the fact that  $y^{LH} \leq y^{LL}$ , implies that  $\tilde{c}^{HH-LH} > \tilde{c}^{LL-LH}$ . From (45), evaluated at  $T_b^L = T_b^H = 0$ , we have that  $c^{LH} - \tilde{c}^{HH-LH} < \Delta b$  and thus:

$$U(\tilde{c}^{HH-LH}, y^{LH}/\omega^H) = U(c^{HH}, y^{HH}/\omega^H) > U(c^{LH} - \Delta b, y^{LH}/\omega^H)$$

meaning that (29) is not binding when  $T_b^L = T_b^H = 0$ .

2. Condition (31) cannot be binding at the optimum with taxes when  $T_b^L = T_b^H = 0$ .

Consider the type  $H$  indifference curve that passes through  $(c^{LL} - \Delta b, y^{LL})$ . Because indifference curves gets further apart when  $y$  increases and using (30), this indifference curve is below the one passing through  $(c^{HH} - \Delta b, y^{HH})$ . Therefore  $HL$  individuals are strictly better off with the allocation  $(c^{HH} - \Delta b, y^{HH})$  than with  $(c^{LL} - \Delta b, y^{LL})$ . In other words, (31) cannot be binding.

3. Condition (32) cannot be binding at the optimum with taxes when  $T_b^L = T_b^H = 0$ .

Using (45) together with  $T_b^L = T_b^H = 0$  implies  $\Delta b \geq c^{LH} - \tilde{c}^{LL-LH}$ , with  $\tilde{c}^{LL-LH}$  defined in (45). Then, recalling that  $y^{LL} \geq y^{LH}$  and using the fact that the type  $H$  indifference curves are flatter than the type  $L$  indifference curves, one must have  $\Delta b > c^{LH} - \tilde{c}^{LL-LH}(H)$ , where  $\tilde{c}^{LL-LH}(H)$  is implicitly defined by:

$$U(\tilde{c}^{LL-LH}(H), y^{LL}/\omega^H) = U(c^{LH}, y^{LH}/\omega^H).$$

We then make use of (31) which implies  $\Delta b \geq c^{LL} - \tilde{c}^{HL-LL}$ , where  $\tilde{c}^{HL-LL}$  is implicitly defined by:

$$U(\tilde{c}^{HL-LL}, y^{LL}/\omega^H) = U(c^{HL}, y^{HL}/\omega^H).$$

We finally note that  $(\tilde{c}^{LL-LH}(H), y^{LH})$  is located to the left of  $(c^{LL}, y^{LL})$  implying  $\Delta b \geq \tilde{c}^{LL-LH}(H) - \tilde{c}^{HL-LH}$ , with  $\tilde{c}^{HL-LH}$  defined accordingly. Combining the two inequalities  $\Delta b > c^{LH} - \tilde{c}^{LL-LH}(H)$  and  $b^H \geq \tilde{c}^{LL-LH}(H) - \tilde{c}^{HL-LH}$  implies  $2\Delta b > c^{LH} - \tilde{c}^{HL-LH}$  and therefore that (32) cannot be binding.

4. Condition (33) cannot be binding at the optimum with taxes when  $T_b^L = T_b^H = 0$ .

When  $T_b^L = T_b^H = 0$ , (25) becomes:

$$U(c^{LH}, y^{LH}/\omega^L) \geq U(c^{LL} + \Delta b, y^{LL}/\omega^L).$$

Combining this condition with (36) and recalling that  $y^{HH} \geq y^{LL}$  leads to the conclusion.

5. Condition (34) cannot be binding at the optimum with taxes when  $T_b^L = T_b^H = 0$ .

Define  $\tilde{c}^{LL-HL}$  as the consumption level that satisfies:

$$U(\tilde{c}^{LL-HL}, y^{HL}/\omega^L) = U(c^{LL}, y^{LL}/\omega^L).$$

Condition (35) implies:

$$\Delta b \leq \tilde{c}^{LL-HL} - c^{HL}. \quad (48)$$

Define  $\tilde{c}^{LL-HL}$  and  $\tilde{c}^{LH-HL}$  as the consumption levels that satisfy respectively:

$$U(\tilde{c}^{LL-HL}, y^{HL}/\omega^L) = U(c^{LL}, y^{LL}/\omega^L)$$

and

$$U(\tilde{c}^{LH-HL}, y^{HL}/\omega^L) = U(c^{LH}, y^{LH}/\omega^L).$$

We know from (46) that  $\Delta b \leq \tilde{c}^{LH-HL} - c^{LL}$ . Because  $y^{HL} \geq y^{LL}$  (it has been proven above that  $y^{HL} \geq y^{HH}$  and  $y^{HH} \geq y^{LL}$ ) and the distance between indifference curves increases with  $y$ , we obtain that  $\Delta b < \tilde{c}^{LH-HL} - \tilde{c}^{LL-HL}$ . Condition (48) then implies  $2\Delta b < \tilde{c}^{LH-HL} - c^{HL}$ , which in turn implies that (34) cannot be binding when  $T_b^L = T_b^H = 0$ .

6. Condition (35) cannot be binding at the optimum with taxes when  $T_b^L = T_b^H = 0$ .

This follows from the observation that  $\tilde{c}^{LL-HL} > \tilde{c}^{HH-HL}$  when (36) is satisfied. Then (35) cannot be binding when (18) is satisfied, as  $\Delta b \leq \tilde{c}^{HH-HL} - c^{HL}$ , which corresponds to inequality (47) when  $T_b^L = T_b^H = 0$ .

7. Condition (36) cannot be binding at the optimum with taxes when (30) is binding.

This is a direct consequence of the fact that indifference curves of the type  $L$  individuals are steeper than the ones of the types  $H$ , combined with our finding that  $y^{HH} \geq y^{LL}$ .



### D.3 Binding bequests constraints at the tax optimum

1. Constraints (39) and (40) are not binding at the optimum with taxes.

We rewrite condition (40) as follows:

$$\begin{aligned}
& U(c^{LL}, y^{LL}/\omega^L) - U(c^{LL} - \Delta b, y^{LL}/\omega^L) \\
+ & \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) \\
- & \gamma(p^L U(c^{LH}, y^{LH}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \geq 0.
\end{aligned}$$

From (21),  $U(c^{HL}, y^{HL}/\omega^H) \geq U(c^{HH} - \Delta b, y^{HH}/\omega^H)$ . Therefore:

$$\begin{aligned}
& U(c^{HL}, y^{HL}/\omega^H) - U(c^{HH}, y^{HH}/\omega^H) \\
\geq & U(c^{HH} - \Delta b, y^{HH}/\omega^H) - U(c^{HH}, y^{HH}/\omega^H) \\
= & u(c^{HH} - \Delta b) - u(c^{HH}).
\end{aligned}$$

Recalling that  $c^{HH} \geq c^{LL}$ , the concavity of the utility function implies  $u(c^{HH} - \Delta b) - u(c^{HH}) \geq u(c^{LL} - \Delta b) - u(c^{LL})$ .

From (28),  $U(c^{LL}, y^{LL}/\omega^L) \geq U(c^{LH} - \Delta b, y^{LH}/\omega^L)$ . Therefore:

$$\begin{aligned}
& U(c^{LL}, y^{LL}/\omega^L) - U(c^{LH}, y^{LH}/\omega^L) \\
\geq & U(c^{LH} - \Delta b, y^{LH}/\omega^L) - U(c^{LH}, y^{LH}/\omega^L) \\
= & u(c^{LH} - \Delta b) - u(c^{LH}).
\end{aligned}$$

Recalling that  $c^{LH} \geq c^{LL}$ , the concavity of the utility function implies  $u(c^{LH} - \Delta b) - u(c^{LH}) \geq u(c^{LL} - \Delta b) - u(c^{LL})$ .

It follows that

$$\begin{aligned}
& \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) \\
- & \gamma(p^L U(c^{LH}, y^{LH}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \\
\geq & \gamma(u(c^{LL} - \Delta b) - u(c^{LL})),
\end{aligned}$$

and thus

$$\begin{aligned}
& U(c^{LL}, y^{LL}/\omega^L) - U(c^{LL} - \Delta b, y^{LL}/\omega^L) \\
& + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) \\
& - \gamma(p^L U(c^{LH}, y^{LH}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \\
& \geq (1 - \gamma)(u(c^{LL}) - u(c^{LL} - \Delta b)) \geq 0.
\end{aligned}$$

We now turn to condition (39), that can be rewritten as follows:

$$\begin{aligned}
& U(c^{LH} - \Delta b, y^{LH}/\omega^L) - U(c^{LH}, y^{LH}/\omega^L) \\
& + \gamma(p^L (U(c^{LH}, y^{LH}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L)) + p^H (U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H))) \\
& \leq 0.
\end{aligned}$$

From (28),  $U(c^{LL}, y^{LL}/\omega^L) \geq U(c^{LH} - \Delta b, y^{LH}/\omega^L)$  and thus

$$U(c^{LH}, y^{LH}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L) \leq u(c^{LH}) - u(c^{LH} - \Delta b).$$

From (21),  $U(c^{HL}, y^{HL}/\omega^H) \geq U(c^{HH} - \Delta b, y^{HH}/\omega^H)$  and thus

$$U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H) \leq u(c^{HH}) - u(c^{HH} - \Delta b).$$

We have shown previously that  $c^{HH} \geq c^{LH}$ . Therefore  $u(c^{HH}) - u(c^{HH} - \Delta b) \leq u(c^{LH}) - u(c^{LH} - \Delta b)$ . All this implies:

$$\begin{aligned}
& U(c^{LH} - \Delta b, y^{LH}/\omega^L) - U(c^{LH}, y^{LH}/\omega^L) \\
& + \gamma(p^L (U(c^{LH}, y^{LH}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L)) + p^H (U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H))) \\
& \leq (1 - \gamma)(u(c^{LH} - \Delta b) - u(c^{LH})) \leq 0.
\end{aligned}$$

2. Conditions (37) and (38) cannot be binding at the optimum with taxes when (30) is binding.

Let us first consider constraint (38). From (20), we have:

$$\begin{aligned}
& U(c^{HL}, y^{HL}/\omega^H) \\
& + \gamma(p^L U(c^{LH}, y^{LH}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \\
& - \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) \\
& \geq U(c^{LL}, y^{LL}/\omega^H).
\end{aligned}$$

Therefore (38) will be satisfied if:

$$U(c^{LL}, y^{LL}/\omega^H) \geq U(c^{HL} + \Delta b, y^{HL}/\omega^H).$$

When (31) binds, we have  $U(c^{LL}, y^{LL}/\omega^H) = U(c^{HH}, y^{HH}/\omega^H)$ . Using (18), (38) must therefore be satisfied.

We then show that (37) is satisfied when (38) is. These two conditions can indeed be rewritten respectively:

$$\begin{aligned}
u(c^{HH}) - u(c^{HH} + \Delta b) & \geq \gamma(\Omega^L - \Omega^H) \\
u(c^{HL}) - u(c^{HL} + \Delta b) & \geq \gamma(\Omega^L - \Omega^H).
\end{aligned}$$

As  $c^{HH} \geq c^{HL}$ , the concavity of the utility function implies that  $u(c^{HH}) - u(c^{HH} + \Delta b) \geq u(c^{HL}) - u(c^{HL} + \Delta b)$  and therefore that (37) is satisfied when (38) is.

## E Proof of proposition 4

The proof consists in showing that (30) is necessarily binding at the tax optimum when  $T_b^L = T_b^H = 0$  and  $\gamma$  and  $p^L$  are close enough to 1. The conclusion then follows from proposition 3.

If (19) does not bind, then (30) is necessarily binding, because otherwise it would be possible to increase social welfare by decreasing both  $c^{HL}$  and  $c^{HL}$ .

Consider now the case where (19) is binding. We write this condition with equality when  $T_b^L = T_b^H = 0$ :

$$U(c^{HH}, \frac{y^{HH}}{\omega^H}) + \gamma\Omega^H = U(c^{LL} + \Delta b, \frac{y^{LL}}{\omega^H}) + \gamma\Omega^L. \quad (49)$$

Observe then that:

$$\begin{aligned}\lim_{\gamma, p^L \rightarrow 1} \Omega^L &= U(c^{LL}, y^{LL}/\omega^L) \\ \lim_{\gamma, p^L \rightarrow 1} \Omega^H &= U(c^{LH}, y^{LH}/\omega^L).\end{aligned}$$

In the remainder of the demonstration, all expressions are computed in the limit case where  $\gamma, p^L \rightarrow 1$ .

Condition (49) can then be rewritten as:

$$U(c^{HH}, \frac{y^{HH}}{\omega^H}) = U(c^{LL} + \Delta b, \frac{y^{LL}}{\omega^H}) + U(c^{LL}, y^{LL}/\omega^L) - U(c^{LH}, y^{LH}/\omega^L). \quad (50)$$

Define  $\tilde{b}$  as follows:

$$U(c^{LH}, y^{LH}/\omega^L) = U(c^{LL} + \tilde{b}, y^{LL}/\omega^L). \quad (51)$$

Condition (25) implies  $\Delta b \leq \tilde{b}$  and therefore  $U(c^{LL} + \Delta b, y^{LL}/\omega^H) \leq U(c^{LL} + \tilde{b}, y^{LL}/\omega^H)$ . This condition, combined with (50) and (51), leads to:

$$U(c^{HH}, \frac{y^{HH}}{\omega^H}) \leq U(c^{LL} + \tilde{b}, \frac{y^{LL}}{\omega^H}) + U(c^{LL}, \frac{y^{LL}}{\omega^L}) - U(c^{LL} + \tilde{b}, \frac{y^{LL}}{\omega^L}).$$

Observing that:

$$\begin{aligned}& U(c^{LL}, \frac{y^{LL}}{\omega^L}) - U(c^{LL} + \tilde{b}, \frac{y^{LL}}{\omega^L}) \\ &= u(c^{LL}) - v(\frac{y^{LL}}{\omega^L}) - u(c^{LL} + \tilde{b}) - v(\frac{y^{LL}}{\omega^L}) \\ &= u(c^{LL}) - u(c^{LL} + \tilde{b}) \\ &= u(c^{LL}) - v(\frac{y^{LL}}{\omega^H}) - u(c^{LL} + \tilde{b}) - v(\frac{y^{LL}}{\omega^H}) \\ &= U(c^{LL}, \frac{y^{LL}}{\omega^H}) - U(c^{LL} + \tilde{b}, \frac{y^{LL}}{\omega^H}),\end{aligned}$$

we obtain:

$$U(c^{HH}, \frac{y^{HH}}{\omega^H}) \leq U(c^{LL}, \frac{y^{LL}}{\omega^H}),$$

meaning that (30) is binding. qed.

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