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## Optimal income taxation with tax avoidance

Georges Casamatta

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Georges Casamatta†

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**Abstract**

We determine the optimal income tax schedule when individuals have the possibility of avoiding paying taxes. Considering a convex concealment cost function, we find that a subset of individuals, located in the interior of the income distribution, should be allowed to avoid taxes, provided that the marginal cost of avoiding the first euro is sufficiently small. This contrasts with the results of Grochulski (2007) who shows that, with a subadditive cost function, all individuals should declare their true income. We also provide a characterization of the optimal income tax curve.

Keywords: fiscal avoidance, optimal income tax.

JEL: H21.

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†UMR CNRS 6240 LISA, Università di Corsica Pasquale Paoli, Corte, France; email: casamatta_g@univ-corse.fr

# 1 Introduction

Individual responses to taxation can be classified into two broad categories. On the one hand, individuals react to taxation by changing arguments of the utility function, i.e. leisure and other goods and services. Slemrod (1995) names this effect the *real* response to taxation. Conceptually distinct from real substitution responses are efforts to reduce one's tax liability without modifying economic decisions, such as labor supply or savings. These responses can be legal (*avoidance*) or not (evasion). Slemrod and Yitzhaki (2002), building on the work of Stiglitz (1985), distinguish three basic principles of tax avoidance: retiming, tax arbitrage and income shifting. Retiming occurs when the timing of certain transactions responds to changes in tax rates. A classic example is the anticipation of capital gains realizations following the announcement of the tax rate increase in the Tax Reform Act of 1986 (TRA86). Tax arbitrage denotes all the activities that take advantage of inconsistencies in the tax law. Income shifting arises when the reduction in reported incomes is due to a shift away from taxable individual income toward other forms of taxable income, such as corporate income. An illustration is given by the shift from C corporations into S corporations (which are taxed like partnerships and therefore are not subject to the corporation income tax) following the drop in the top individual rate below the corporate rate in TRA86.

There exists now quite a substantial empirical literature, summarized in Saez et al. (2012), that assesses the extent of avoidance responses to taxation. These studies are mainly based on the natural experiment provided by TRA86. Saez (2004) finds that income shifting can explain most of the rise in Subchapter S and partnership income. Gruber and Saez (2002) estimate and compare the elasticities of taxable and of broad income. They find a much lower value for the former, suggesting that much of the taxable income response comes through deductions, exemptions, and exclusions. Kleven et al. (2011) conduct a field experiment and determine the effects of changes in marginal tax rates on reported income. They conclude that most of the elasticity of reported income with respect to tax rates can be explained by (legal) avoidance rather than (illegal) evasion. Overall there is compeling evidence of strong behavioral responses to taxation. Moreover these responses fall mainly in the avoidance category.

On the theoretical side, the optimal taxation literature, initiated by Mirrlees (1971), has mainly focused on the real responses to taxation. It aims at identifying the optimal income tax curve when individuals react to the tax by decreasing their labor supply. As argued before, this response is not the empirically most relevant. The taxable income is very sensitive to the tax rate mainly because of tax avoidance and evasion.[1]

Slemrod (2001) studies the effect of income taxation in a model where both real (change in labor supply) and avoidance responses are taken into account. He does however adopt a purely positive standpoint and does not determine the optimal level of taxes. Slemrod and Kopczuk (2002) determine the optimal level of avoidance. Contrarily to labor supply responses, avoidance behaviors can be, at least partly, controled by the government. This has crucial implications for the design of the tax system. If avoidance responses to taxation are large, the best policy would not be to lower tax rates (as suggested by the standard Mirrleesian approach), but instead to broaden the tax base and eliminate avoidance opportunities.

Few theoretical studies address the problem of the optimal nonlinear income tax schedule when individuals try to avoid taxes, with the notable exception of Grochulski (2007).[2] This latter develops a standard optimal taxation model, in which individuals respond to the income tax by hiding part of their income, at a cost, instead of reducing their labor supply, as in the Mirrlees model. This setting constitutes an application of the costly state verification literature (Lacker and Weinberg (1989)) to optimal taxation. He finds two main results. First, at the optimum with taxes, no individuals should hide income. This result is called the no-falsification theorem. Second, the optimal tax schedule is such that marginal tax rates are equal to the marginal falsification costs.

These results are derived with a subadditive concealment cost function. In this article, we consider a convex cost function (that violates subadditivity). We are not aware of any empirical study that determine the shape of the cost function. We thus believe that studying the convex case is at least of theoretical interest. We are moreover convinced that it fits the real-world to a large extent: as the amount to be avoided becomes large, it should become harder and harder to find profitable avoidance opportunities.

It turns out that, in this setting, the no-falsification theorem does not hold anymore. We show that, provided that the marginal cost of concealing the first euro is low enough,

[1] For theoretical studies of the optimal tax schedule when individuals evade taxes, see Cremer and Gahvari (1996) or Chander and Wilde (1998).

[2] Piketty et al. (2014) determine the optimal tax for top income individuals in the presence of avoidance, which amounts to solve an optimal linear taxation problem. Landier and Plantin (2017) extend the setup of Grochulski (2007) to an environment in which income generation involves risk-taking.

individuals belonging to the middle-class should optimally hide part of their income to the fiscal authority. For a marginal cost close to 1 however, all individuals should declare their true income. Finally the first-best (that consists in fully equalizing after-tax incomes) is achieved when the marginal cost is large enough (equal to 1). We also characterize optimal marginal tax rates and thus the shape of the optimal income tax schedule. Marginal tax rates are constant for non-avoiding people. They are greater for individuals who avoid paying taxes. The way they vary with income depend on the shape of the income distribution, as well as the characteristics of the concealment cost function and the preferences of the social planner. We construct an example leading to a bell-shaped curve of optimal tax rates. The corresponding optimal tax schedule is first convex and then concave.

In the last section, we study some extensions of the basic model. We first discuss some generalizations of the cost function (cost varying with income, fixed cost of avoidance). We then develop a two-types model with both labor supply and avoidance responses. This simple model suggests that the main results extend to the case of endogenous labor supply.

# 2 Model

Individuals differ with respect to income $w$, distributed according to the cumulative distribution function $F(.)$ and the density $f(.)$ on the support $[w_-, w_+]$; average income is denoted $\overline{w}$. Labor supply is assumed to be inelastic so that income is fixed.[3]

True income is not observable to the fiscal authority and individuals have the possibility to hide (legally) part of it to the government. This action is however costly and we denote $\phi(\Delta)$ the cost of hiding $\Delta$ euros, with $\phi(0) = 0$. We allow for the possibility that individuals declare more than their true income, in which case $\Delta < 0$. We assume that $\phi$ is continuous and strictly convex.[4] Moreover, it is decreasing for $\Delta < 0$ and increasing for $\Delta > 0$. It is differentiable everywhere, except at 0 where the right-hand (resp. left-hand) derivative is positive (resp. negative).[5] We finally assume that the marginal cost of hiding the first euro is not too high: $\phi'(0) \leq 1$.

The utility function depends only on consumption $c$, i.e. after-tax income. We assume

[3]See section 4.2.2 for an analysis with endogenous labor supply.

[4]Strict convexity, combined with the fact that $\phi(0) \leq 0$, implies that $\phi$ violates the subaddivity assumption in Grochulski (2007) and Landier and Plantin (2017), where a function $f$ is subadditive iff $f(x + y) \leq f(x) + f(y)$.

[5]The right- and left-hand derivatives coincide when $\phi'(0) = 0$.

for simplicity a linear utility function: $u(c) = c$.

# 3 The optimal income tax schedule

## 3.1 The social planner's program

The social planner designs a contract that specifies a tax level contingent on the income displayed by individuals (which does not necessarily corresponds with their true income). Lacker and Weinberg (1989) show that the revelation principle applies in this framework. The optimal contract is therefore achieved through a direct and revealing mechanism, such that that individuals report directly and truthfully their type (their true income) to the social planner. According to this mechanism, an individual with true income $w$ should display income $r(w)$ and pays a tax $T(r(w))$ on this income.

The social planner determines the functions $r(.)$ and $T(.)$ that maximize social welfare, expressed as the sum of a concave transformation $G(.)$ of individual utility levels, under resource and incentive constraints:

$$(P1) \max_{r(.),T(.)} \int G(U(w))dF(w)$$

st

$$U(w) = w - T(r(w)) - \phi(w - r(w)), \tag{1}$$

$$\int T(r(w))f(w)dw \geq 0 \tag{2}$$

and

$$U(w) \geq w - T(r(w')) - \phi(w - r(w')),\ \forall w, w' \in [w_-, w_+]. \tag{3}$$

The utility level of an individual with income $w$ is expressed in equation (1) (recall that the utility function is linear in consumption). Equation (2) represents the Government Budget Constraint (GBC) and (3) is the incentive constraints: a type $w$ individual should not want to pretend that he is of type $w'$.

The incentive constraint implies that every individual should report truthfully his type. Therefore:

$$w = \arg\max_{w'} w - T(r(w')) - \phi(w - r(w')),$$

The first-order condition then implies:

$$-r'(w)T'(r(w)) + r'(w)\phi'(w - r(w)) = 0. \tag{4}$$

and thus:

$$T'(r(w)) = \phi'(w - r(w)). \tag{5}$$

In words, the marginal tax rate should be equal to the marginal avoidance cost. This result, which has already been obtained by Slemrod (2001) and Grochulski (2007) is intuitive: should the marginal tax rate be lower (resp. greater) than the marginal cost, individuals should decrease (resp. increase) the amount of avoidance. A second lesson of this formula is that, because the cost function is assumed to be convex, *individuals who conceal more income face larger marginal tax rates.*

Using standard technique in mechanism design (Fudenberg and Tirole (1991), chapter 7), the second-order condition for a local optimum can be shown to be:

$$r'(w)\phi''(w - r(w)) \geq 0.$$

As the cost function is assumed to be strictly convex, the second-order condition is satisfied if and only if $r'(w) \geq 0$, i.e. reported income increases with true income.[6] Violation of this condition implies that a subset of individuals should be bunched at the same allocation, declaring the same level of income and paying the same amount of taxes. In the remainder of this article, we shall assume that the second-order condition is satisfied.[7]

Recalling that $U(w) = w - T(r(w)) - \phi(w - r(w))$ and using (4), we have:

$$\frac{dU}{dw} = 1 - \phi'(w - r(w)). \tag{6}$$

This condition is intuitive. The social planner, who wants to equalize utility levels in the first-best (see section 3.2), wishes to make the variation in utility with respect to income as small as possible. Incentive constraints however prevent him from achieving this allocation. If the second-best allocation were to imply $dU/dw < 1 - \phi'(w - r(w))$, it would not be incentive compatible as the individual $w$ would want to mimic the individual with a little less income. Differentitating (1), is is easily seen that $dU/dw < 1-\phi'(w-r(w))$ is equivalent to $T'(r(w)) > \phi'(w - r(w))$. Therefore, it would be welfare improving for the individual $w$ to under-declare his income: the decrease in tax liability, $T'(r(w))$, would more than compensate the savings on avoidance costs, $\phi'(w - r(w))$.

[6]This is the analogous condition to having pre-tax income being increasing with productivity in the optimal taxation literature (Theorem 1 in Mirrlees (1971)).

[7]For a careful treatment of bunching in optimal taxation models, see Lollivier and Rochet (1983), Ebert (1992) or Boadway et al. (2000).

Using condition (6), we can restate the planner's problem as follows:

$$(P2) \max_{r(.),T(.)} \int G(U(w))dF(w)$$

st

$$U(w) = w - T(r(w)) - \phi(w - r(w)),$$
$$\int T(r(w))f(w)dw \geq 0,$$
$$\frac{dU}{dw} = 1 - \phi'(w - r(w)).$$

### 3.2 The solution without incentive constraints: first-best allocation

Without incentive constraints, there is no cost in making individuals reveal their true income, so that the first-best allocation can be achieved. Solving program (P1) without the constraints (3) and denoting $\mu$ the Lagrange multiplier of the GBC, we get:

$$G'(U(w)) = \mu.$$

Because the government maximizes a concave transformation of individual utilities, the first-best allocation consists in giving all individuals the same consumption level. Moreover, because avoidance is costly, no individual should falsify his income. This implies that all individuals consume the average income in society, as summarized in the next proposition:

**Proposition 1.** *At the first-best allocation, consumption is equalized across individuals who all consume the average income: $c(w) = \overline{w}\ \forall w$. This allocation can be implemented with a linear tax schedule having a 100% marginal tax rate and a lump-sum transfer equal to $\overline{w}$.*

It is easily seen that the first-best allocation is not incentive-compatible, as soon as the marginal cost of avoiding the first euro is strictly less than 1. To implement the first-best, one must have a tax schedule with 100% marginal tax rates. Avoiding 1 euro at the margin then increases consumption by the same amount, less the avoidance cost. As soon as this cost is lower than 1, it is thus optimal for individuals to conceal part of their income.

### 3.3 Optimal allocation under no-falsification

Before turning to the general case, we determine the optimal allocation constrained by the fact that individuals do not falsify their income.

Suppose that all individuals declare their true income: $r(w) = w$, $\forall w$. Then (6) implies:

$$\frac{dU}{dw} = 1 - \phi'(0).$$

Integrating this condition yields:

$$U(w) = (1 - \phi'(0))w + k,$$

Recalling that utility is equal to consumption, the GBC (2) can be written:

$$\int U(w)f(w)dw = \overline{w} - \int \phi(w - r(w))f(w)dw.$$

As $r(w) = w$ and $\phi(0) = 0$, this becomes:

$$\begin{aligned} & \int U(w)f(w)dw = \overline{w} \\ \Leftrightarrow \quad & (1 - \phi'(0))\overline{w} + k = \overline{w} \\ \Leftrightarrow \quad & k = \overline{w}\phi'(0). \end{aligned}$$

Hence the next proposition:

**Proposition 2.** *The optimal allocation when no individual avoids taxes is such that:* $c(w) = (1 - \phi'(0))w + \overline{w}\phi'(0)$*. This allocation can be implemented with a linear tax schedule having a marginal tax rate equal to* $\phi'(0)$ *and a lump-sum transfer equal to* $\overline{w}\phi'(0)$*.*

This result is related to the discussion in the previous section: as long as $\phi'(0) < 1$, incentive constraints prevent the first-best from being implemented. When $\phi'(0) = 1$, we can conclude from this proposition that the first-best can be implemented: no individual avoids and everyone gets the same consumption level.

We have determined the optimal allocation when no individual avoid taxes. This allocation could however be improved by allowing some individuals to falsify their income. We treat this question in the next section.

## 3.4 The optimality of avoidance

To know in which circumstances the possibility of avoiding taxes increases welfare, we solve program (P2) above.

Taking $U(w)$ as the state variable and $r(w)$ as the control variable, we form the Hamiltonian associated to program (P2):[8]

$$\mathcal{H} = (G(U(w)) + \mu T(r(w)))f(w) + \lambda(w)\frac{dU}{dw},$$

[8] This formulation is standard in the optimal taxation literature. See for example Stiglitz (1987).

where $\mu$ and $\lambda(w)$ are the multipliers associated to the GBC and the incentive constraints respectively. The first-order conditions for interior solutions are then

$$
\begin{aligned}
&\frac{\partial \mathcal{H}}{\partial r} = 0 \\
&\Leftrightarrow \mu \left.\frac{dT}{dr}\right|_U f(w) + \lambda(w)\phi''(w - r(w)) = 0, &(7)\\
&\frac{\partial \mathcal{H}}{\partial U} = -\lambda'(w) \\
&\Leftrightarrow -\lambda'(w) = (G'(U(w)) + \mu \left.\frac{dT}{dU}\right|_r )f(w). &(8)
\end{aligned}
$$

For given $U$ and $r$, $T$ is obtained from (1). Differentiating this equation, we get $dT/dr|_U = \phi'(w - r(w))$ and $dT/dU|_r = -1$, so that conditions (7) and (8) become:

$$
\mu\phi'(w - r(w))f(w) + \lambda(w)\phi''(w - r(w)) = 0, \qquad (9)
$$
$$
-\lambda'(w) = (G'(U(w)) - \mu)f(w).
$$

Integrating the second condition and using the endpoint condition $\lambda(w_+) = 0$ yields

$$
-\lambda(w) = \int_w^{w_+} (-G'(U(t)) + \mu)f(t)dt. \qquad (10)
$$

The multiplier $\lambda$ measures the change in social welfare when individuals from $w$ to the top are taken one extra euro. On the one hand, the utility of the concerned individuals decreases: this is valued $-G'(U(t)))$ by the social planner. On the other hand, this change yields additional revenue to society; the corresponding change in social welfare is given by $\mu$, the multiplier of the GBC.

From the endpoint condition $\lambda(w_-) = 0$, we obtain:

$$
\mu = \int G'(U(w))dF(w). \qquad (11)
$$

Recalling that $U$ is increasing with $w$ (see (6)) and inspecting (10) and (11), it should be observed that $\lambda(w)$ is negative.[9] Note also that $\lambda(w_-) = \lambda(w_+) = 0$ and thus there is no benefit of relaxing the incentive constraint at both ends of the income distribution. Relaxing the incentive constraint at $w_+$ yields no benefit since there are no people to tax above this level. In a symmetrical way, relaxing the incentive constraint at $w_-$ allows to tax people above that level. However the only beneficiaries are those with income $w_-$, who have zero

[9] Relaxing the incentive constraint at $w$ allows to increase the tax paid by all individuals with income above $w$, this tax increase being redistributed to people with income below $w$. This benefit is measured by $-\lambda(w)$.

mass in the continuous case under study (Seade (1977)). This explains the difference with the finite-types model in which the lowest types do not constitute a negligible proportion of the population (Stiglitz (1987)).

We now establish a first feature of the optimal contract: people should not overstate their income level.

**Proposition 3.** *At the optimal (second-best) allocation: $r(w) \leq w$.*

*Proof.* We evaluate the derivative of the Hamiltonian when $r(w)$ approaches $w$ from the right:

$$\left.\frac{\partial \mathcal{H}}{\partial r}\right|_{r(w)\searrow w} = \mu \phi'_{-}(0) f(w) + \lambda(w)\phi''_{-}(0) < 0,$$

where $\phi'_{-}$ and $\phi''_{-}$ denote the left-hand first and second derivatives of $\phi$ respectively. By assumption, $\phi'_{-}(0) \leq 0$ and $\phi''_{-}(0) > 0$. Moreover, we have established previously that $\lambda(w) \leq 0$. Therefore when $r(w)$ is larger than $w$, it is welfare-improving to decrease it. □

It follows that individuals either declare their true income or understate their income to the fiscal administration. Claiming to have more income then what is really earned is at the same time costly and make incentive constraints more stringent (as represented by the negative $\lambda$).

We now argue that, when the marginal cost of hiding the first euro, $\phi'(0)$, is low enough, some individuals, located in the interior of the income distribution, will report strictly less than their true income. On the other hand, for $\phi'(0)$ sufficiently close to 1, all individuals report truthfully their income and there is no tax avoidance at the optimum. In such a case, we recover the tax schedule described in proposition 2. These results are summarized in the following proposition.

**Proposition 4.** *1. There exist $\phi_1$ and $\phi_2$, where $0 < \phi_1 \leq \phi_2 < 1$, such that*

*(i) If $\phi'(0) \geq \phi_2$, $r(w) = w$, $\forall w$;*

*(ii) If $\phi'(0) < \phi_1$, $\exists w \in (w_-, w_+)$ such that $r(w) < w$.*

*2. When $\phi'(0) < \phi_1$, there exist $w_{\inf} > w_-$ and $w_{\sup} < w_+$ such that*

*(i) $r(w) = w$, $\forall w \leq w_{\inf}$ and $w \geq w_{\sup}$;*

*(ii) There exists $\delta > 0$ such that $r(w_{\inf} + \delta) < w_{\inf} + \delta$ and $r(w_{\sup} - \delta) < w_{\sup} - \delta$.*

*Proof.* see appendix □

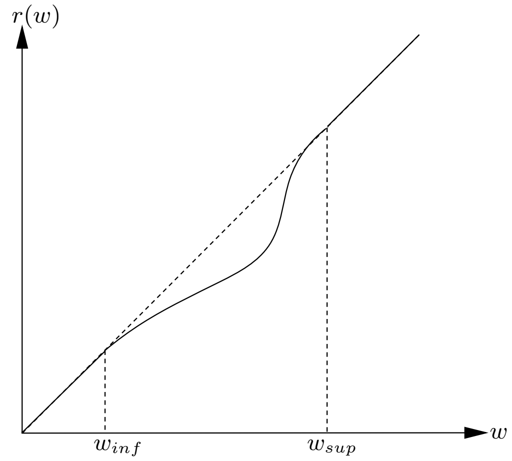


Figure 1: Reported incomes

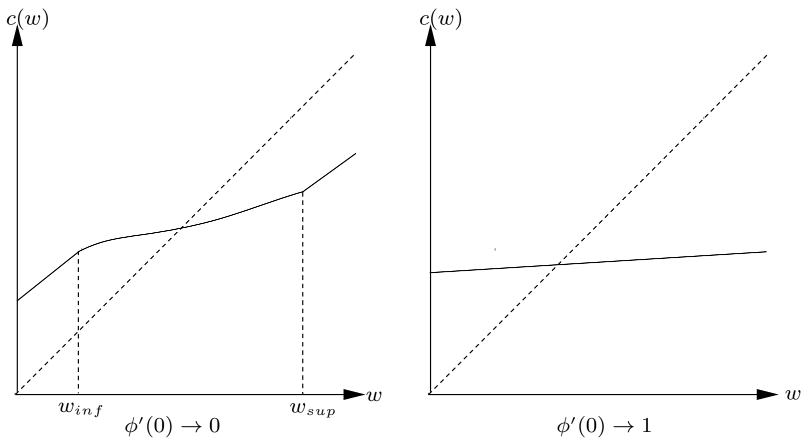


Figure 2: Consumption levels

Optimal reported incomes and consumption levels are represented in figures 1 and 2 respectively.

We now give the intuition of our main result, namely that some individuals should optimally conceal income when $\phi'(0) \to 0$. Suppose there is no avoidance and make individual $\tilde{w}$ avoid at the margin by perturbing the consumption schedule as represented on figure 3 (the black line is the initial consumption schedule and the blue curve the perturbed one). If this new consumption schedule is both feasible and incentive compatible, it is then socially preferred to the original one (as it allows to "flatten" the consumption curve), meaning that avoidance is optimal.

Making $\tilde{w}$ avoid at the margin ($r(w) = \tilde{w} - \varepsilon$) allows to relax incentive constraints:

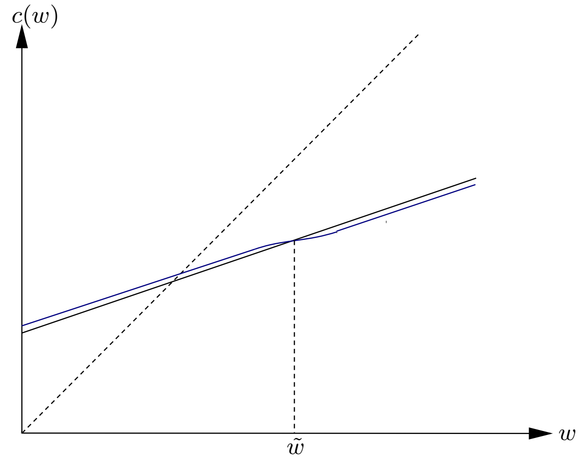


Figure 3: The effect of allowing avoidance

because of convex concealment costs, higher income individuals are less tempted to mimic $\tilde{w}$.[10] This corresponds to the term $-\lambda(w)\phi''(0)$ in (9). But it also has a cost represented by the term $\mu\phi'(0)f(w)$: $\tilde{w}$ must incur a lower tax in order to stay at the same consumption level (to be compensated for the cost of avoidance). When $\phi'(0) \to 0$, the benefit outweighs the cost for almost all individuals (not for individuals at the extreme of the distribution as $\lambda(w_-) = \lambda(w_+) = 0$). When $\phi'(0) \to 1$, $\lambda(w) \to 0$ and the cost outweighs the benefit for all individuals. It thus explains why it is optimal to allow for avoidance when the marginal cost of concealing the first euro is low enough. It also helps to explain why it concerns individuals belonging to the middle-class and not the very poor and the very rich.

In the standard Mirrlees models, the allocation of all types is distorted (except possibly $w_-$ and $w_+$). This comes from the fact that the cost of creating "small" distortions is negligible and thus the redistributive benefit outweighs the cost for all types in the interior of the distribution. Here the cost of distortions, measured by $\phi'(0)$, is not negligible except in the particular case $\phi'(0) = 0$. This explains the difference with the Mirrlees model.

[10] To see this, consider a discretized version of the model and assume that individuals $\tilde{w} + \delta$ are indifferent between mimicking $\tilde{w}$ or not:

$$\tilde{w} + \delta - T(r(\tilde{w} + \delta)) = \tilde{w} + \delta - T(r(\tilde{w})) - \phi(\delta).$$

Now suppose that $\tilde{w}$ avoid taxes by declaring $\tilde{w} - \varepsilon$ instead of $\tilde{w}$. The utility from complying for individuals $\tilde{w} + \delta$ is unchanged (it is equal to $\tilde{w} + \delta - T(r(\tilde{w} + \delta))$). However, the utility when mimicking is now $\tilde{w} + \delta - T(r(\tilde{w} - \varepsilon)) - \phi(\delta + \varepsilon)$. The change in the tax paid is thus $T(r(\tilde{w} - \varepsilon)) - T(r(\tilde{w}))$ while the change in avoidance cost is $\phi(\delta + \varepsilon) - \phi(\delta)$. For $\varepsilon$ small enough, these changes can be approximated by $T'(r(\tilde{w}))$ and $\phi'(\delta)$ respectively. Because the cost function is convex, we have that the increase in the avoidance cost $\phi'(\delta)$ is larger than the savings in taxes $T'(r(\tilde{w})) \approx \phi'(0)$. Therefore $\tilde{w} + \delta$ is not indifferent anymore and strictly prefers not to mimic $\tilde{w}$. In other words, the incentive constraint has been relaxed.

### 3.5 Marginal tax rates

From (5), we know that marginal tax rates are equal to marginal avoidance costs and are thus everywhere *positive*. We are however not able to conclude about whether they are lower or greater than 1.

For individuals who declare their true income ($r(w) = w$), we readily obtain that they face the marginal tax $\phi'(0)$. For the others, we can, using (9), express the marginal tax rate as follows:

$$T'(r(w)) = -\frac{\lambda(w)}{\mu}\frac{1}{f(w)}\phi''(w - r(w)).$$

This expression is close to (9) in Diamond (1998) and its interpretation is by now standard in the optimal taxation literature (see, e.g., Saez (2001)). On the one hand, increasing the marginal tax rate at a given income level generates a distortion at this point so that the more there are people at this income level, as measured by $f(w)$, the lower the marginal tax rate should be. The distortion comes from the fact that individuals will react to the increased marginal tax rate by reducing their reported income. The term $1/\phi''(w - r(w))$ measures this distortion (it can be obtained by differentiating (5)) and accordingly the lower $\phi''(.)$, the lower should be the marginal tax rate. On the other hand, raising the marginal tax rate locally allows to raise additional taxes on all individuals with higher income, without affecting incentive constraints. The net benefit of doing so is given by $-\lambda(w)$ (it is divided by $\mu$ in order to convert it from welfare to monetary units). The larger this benefit, the larger the marginal tax rate.

It is thus quite hard to predict how marginal tax rates should vary with income. It depends on the way $\lambda(w)$, $f(w)$ and $\phi''(w - r(w))$ vary with $w$. We should however notice that *marginal tax rates are always larger for individuals who avoid with respect to non-avoiding people.* This is obtained readily by inspecting (5) and observing that, due to the convexity of $\phi$, $\phi'(w - r(w)) > \phi'(0)$ whenever $r(w) < w$.

### 3.6 Numerical illustration

To illustrate the model, we have constructed two numerical examples. In both examples, income is distributed uniformly on the support $[0, 10]$. The cost of avoidance is $\phi(x) = x^2/2 + \alpha x$ (so that $\phi'(0) = \alpha$ and $\phi''(x) = 1$) and $G(x) = \ln x$. In the first simulation, $\alpha = 0.2$ and $\alpha = 0.5$ in the second one. We obtain that, in both simulations, some individuals avoid,

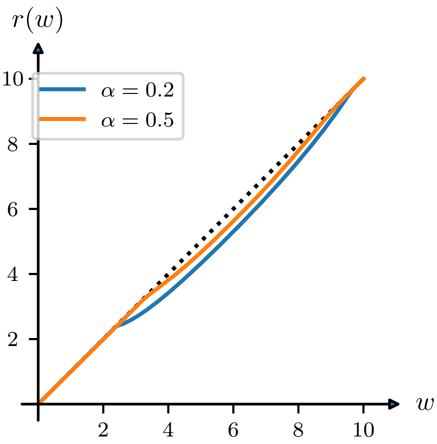


Figure 4: Reported incomes in the numerical examples

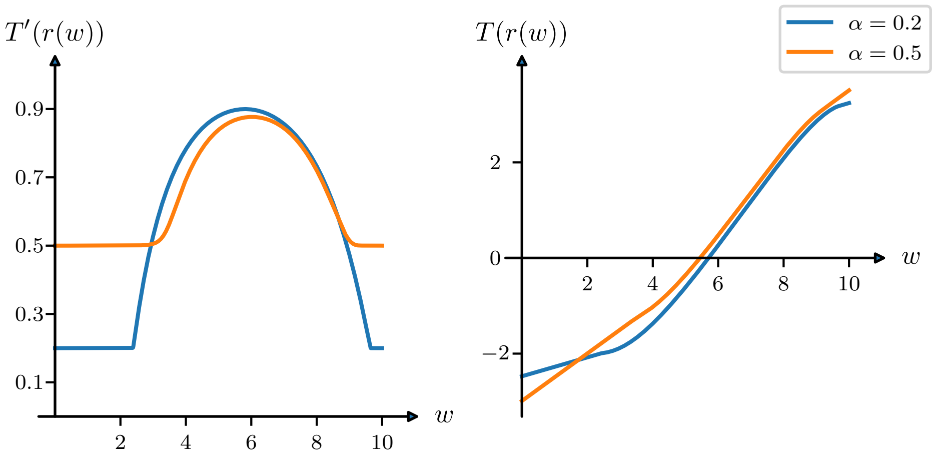


Figure 5: Optimal tax schedule in the numerical examples

the threshold values for the avoiding individuals being respectively $w_{\text{inf}} = 2.41$, $w_{\text{sup}} = 9.60$ and $w_{\text{inf}} = 3.11$, $w_{\text{sup}} = 9.04$. Not surprisingly the set of avoiding people shrinks when the marginal cost $\phi'(0)$ is increased. We also obtain a bell-shaped curve of marginal tax rates, the corresponding optimal tax schedule being first convex and then concave. This is represented in figures 4 and 5.

# 4 Extensions

## 4.1 Generalizing the cost function

Two modifications of the cost function are envisaged. First, we introduce a *fixed* cost in the avoidance technology. Individuals who want to avoid taxes should go through a costly information acquisition process concerning the tax law and have to pay a fixed amount, independently of the amount of income concealed. Second, individuals may have different avoidance opportunities, depending on their income level. In particular richer people may find it easier (meaning incurring a lower total and marginal costs) to avoid taxes than poor individuals. The generalized cost function thus takes the form:

$$\phi(\Delta, w) = \beta(w) + \xi(\Delta, w),$$

where $\beta$ represents the fixed cost (possibly dependent on the true income level) and $\xi$ the variable cost, that both depends on the amount concealed and the true income level.

With this cost function, formula (5) is modified to:

$$T'(r(w)) = \xi_{\Delta}(w - r(w), w).$$

Marginal tax rates are still equal to marginal avoidance costs, but these latter now depend on the income level $w$. If we assume that the marginal cost of avoidance decreases with income ($\xi_{\Delta w} < 0$), this implies that, for a given amount of avoidance, *rich individuals face a lower marginal tax rates than the poor.*

We then turn to the incidence of introducing a fixed cost in the analysis. This modification makes the cost function discontinuous at 0. This in turn implies that the social planner problem is non continuous and cannot be simply solved by analyzing first-order conditions. It is clear that with prohibitive fixed costs, no individuals will be allowed to avoid taxes. With moderate fixed costs however the optimality conditions remain the same as the ones derived above. We conjecture that the main change in the results would be that, intuitively, less individuals avoid taxes. To see this, consider the individuals for which the (unconstrained) solution was $r(w) = w$. In such a case, the planner is indifferent between letting these people avoid at the margin or not. With a fixed cost of avoidance however, the planner now strictly prefers that these individuals declare their true income (as the fixed cost is saved when people do not conceal income). This suggests that *the set of avoiding individuals should shrink when avoidance generates a fixed cost.*

## 4.2 The two-types model

In this section, we consider a simplified two-types model. Considering first the fixed income framework, we show that our results are still valid in this discrete setting and thus do not depend on the incomes being distributed continuously. In a second step, we allow labor supply to be endogenously determined. Here again, we find that our results are robust to this extension.

### 4.2.1 Fixed incomes

We consider two levels of income: $w^L < w^H$, with $n^L$ and $n^H$ the respective numbers of individuals endowed with these income levels.

The (utilitarian) planner chooses bundles $(T^L, \hat{w}^L)$ and $(T^H, \hat{w}^H)$ in order to solve the following program:

$$\max_{T^h, \hat{w}^h} n^L G(w^L - T^L - \phi(w^L - \hat{w}^L)) + n^H G(y^H - T^H - \phi(w^H - \hat{w}^H))$$

st

$$n^L T^L + n^H T^H \geq 0$$

$$w^H - T^H - \phi(w^H - \hat{w}^H) \geq w^H - T^L - \phi(w^H - \hat{w}^L).$$

The first constraint is the GBC and the second one the self-selection constraint. This constraint means that the high types should not select the bundle intended for the low types.[11]

Denoting $\mu$ and $\lambda$ the Lagrange multipliers of the GBC and the incentive constraint respectively, first-order conditions for interior solutions with respect to $T^L$, $T^H$,$\hat{w}^L$, $\hat{w}^H$ respectively read as:

$$\begin{aligned}
-n^L G'(c^L) + \mu n^L + \lambda &= 0 \quad (12)\\
-n^H G'(c^H) + \mu n^H - \lambda &= 0 \quad (13)\\
n^L \phi'(w^L - \hat{w}^L) G'(c^L) - \lambda \phi'(w^H - \hat{w}^L) &= 0 \quad (14)\\
n^H \phi'(w^H - \hat{w}^H) G'(c^H) + \lambda \phi'(w^H - \hat{w}^H) &= 0. \quad (15)
\end{aligned}$$

It is clear that the last condition cannot be satisfied with equality, as the left-hand side is strictly positive. This implies that $w^H = \hat{w}^H$: *highly productive individuals should not avoid*

[11] One can show that the incentive constraint from the low to the high types does not bind under a utilitarian objective.

*taxes at the second-best optimum.*

As for the types $L$, we can deduce easily from the third condition that low productivity individuals should be allowed to avoid taxes as soon as the marginal cost of avoiding the first euro, $\phi'(0)$, is low enough. In this case, allowing avoidance at the margin does not hurt much the types $L$, as the marginal cost $\phi'(0)$ is low. But it allows to relax the incentive constraint, as represented by the term $\lambda\phi'(w^H - \hat{w}^L)$.

We now investigate whether it is possible that the low types declare their true income. In this purpose, we specify functional forms for the functions $G$ and $\phi$: $G(c) = \ln(c)$ and $\phi(\Delta) = \alpha\Delta + \Delta^2/2$, $\forall\Delta \geq 0$. Note that $\phi'(0)$ is equal to $\alpha$. To simplify matters, we also assume that $n^L = n^H \equiv n$. The GBC then becomes $T^L = -T^H$. Subtracting (12) from (13), we get:

$$\begin{aligned} \lambda &= \frac{n(G'(c^L) - G'(c^H))}{2} \\ &= \frac{n}{2}\frac{c^H - c^L}{c^L c^H}. \end{aligned}$$

Evaluating the left-hand side of (14) at $\hat{w}^L = w^L$:

$$\begin{aligned} & n\phi'(0)\tfrac{1}{c^L} - \tfrac{n}{2}\tfrac{c^H - c^L}{c^L c^H}\phi'(w^H - w^L) \\ = & \quad \alpha - \tfrac{c^H - c^L}{2c^H}(\alpha + w^H - w^L) \\ = & \quad \alpha - \tfrac{1}{2}(1 - \tfrac{w^L - T^L}{w^H - T^H})(\alpha + w^H - w^L) \\ > & \quad \tfrac{1}{2}(\alpha - (w^H - w^L)). \end{aligned}$$

As soon as $w^H$ is sufficiently close to $w^L$, the right-hand side is positive when $\alpha > 0$. This means that the no-avoidance result at the bottom of the income distribution is not an artifact of the model with a continuum of types.

### 4.2.2 Endogenous labor supply

We now consider an economy with two goods: leisure $l$ and one consumption good. Individuals differ in productivity: $\omega^L < \omega^H$, with respective numbers $n^L$ and $n^H$. We denote $y$ true labor income and $\hat{y}$ taxable labor income.

We write the utility function as follows:

$$\begin{aligned} U^h &= u(c^h, l^h) \\ &= u(c^h, y^h/\omega^h), \end{aligned}$$

where $c^h$ is the consumption of a type $h$ individual and $l^h$ his labor supply.

We want to determine the income tax schedule that maximizes social welfare. The problem is solved in two steps. In the first stage, the planner determines the amount of taxable income $\hat{y}^h$ and the level of the tax $T^h$ for each type of individual. Then individuals choose optimally, in the second stage, their labor supply (or equivalently pre-tax income). We proceed backward and solve first the program of the individuals in the second stage:

$$\begin{aligned} & \max_{y^h} u(c^h, y^h/\omega^h) \\ & \text{st} \\ c^h &= y^h - T^h - \phi(y^h - \hat{y}^h). \end{aligned}$$

The first-order condition is:

$$(1-\phi')u_c^h + \frac{1}{\omega^h}u_l^h = 0.$$

We substitute the individual budget constraint in the utility function, to obtain the indirect utility function:

$$V^h(\hat{y}^h, T^h) = u(y^h - T^h - \phi(y^h - \hat{y}^h), y^h/\omega^h).$$

Differentiating this function and using the envelop theorem, we get:

$$\frac{\partial V^h}{\partial \hat{y}^h} = \phi'(y^h - \hat{y}^h)u_c^h, \;\; \frac{\partial V^h}{\partial T^h} = -u_c^h.$$

We also write the corresponding expressions for the mimicking individual. When a type $H$ mimics a type $L$, he gets the utility level:

$$\begin{aligned} \tilde{V}^H &= V^H(\hat{y}^L, T^L) \\ &= u(\tilde{c}^H, \tilde{y}^H/\omega^H) \\ &= u(\tilde{y}^H - T^L - \phi(\tilde{y}^H - \hat{y}^L), \tilde{y}^H/\omega^H), \end{aligned}$$

where $\tilde{y}^H$ is the income level that maximizes the utility of type $H$ individuals when the latter declare $\hat{y}^L$ and pay the income tax $T^L$. We thus have:

$$\frac{\partial \tilde{V}^H}{\partial \hat{y}^L} = \phi'(\tilde{y}^H - \hat{y}^L)\tilde{u}_c^H, \;\; \frac{\partial \tilde{V}^H}{\partial T^L} = -\tilde{u}_c^H.$$

We now address the program of the (utilitarian) social planner in the first stage. It chooses bundles $(T^L, \hat{y}^L)$ and $(T^H, \hat{y}^H)$ in order to solve the following program:

$$\max_{T^h, \hat{y}^h} n^L u(y^L - T^L - \phi(y^L - \hat{y}^L), y^L/\omega^L) + n^H u(y^H - T^H - \phi(y^H - \hat{y}^H), y^H/\omega^H)$$

st

$$n^L T^L + n^H T^H \geq 0$$

$$u(y^H - T^H - \phi(y^H - \hat{y}^H), y^H/\omega^H) \geq u(\tilde{y}^H - T^L - \phi(\tilde{y}^H - \hat{y}^L), \tilde{y}^H/\omega^H).$$

Denoting $\mu$ and $\lambda$ the Lagrange multipliers of the GBC and the incentive constraint respectively, the first-order conditions for interior solutions with respect to $T^L$, $T^H$, $\hat{y}^L$, $\hat{y}^H$ respectively read as:

$$\begin{aligned}
-n^L u_c^L + \mu n^L + \lambda \tilde{u}_c^H &= 0 \\
-n^H u_c^H + \mu n^H - \lambda u_c^H &= 0 \\
n^L \phi'(y^L - \hat{y}^L) u_c^L - \lambda \phi'(\tilde{y}^H - \hat{y}^L) \tilde{u}_c^H &= 0 \\
n^H \phi'(y^H - \hat{y}^H) u_c^H + \lambda \phi'(y^H - \hat{y}^H) u_c^H &= 0.
\end{aligned}$$

It is clear that the last condition cannot be satisfied with equality, as the left-hand side is strictly positive. This implies that $y^H = \hat{y}^H$: *highly productive individuals should not avoid taxes at the second-best optimum.*

As for the types $L$, we can deduce from the third condition that low productivity individuals should be allowed to avoid taxes as soon as the marginal cost of avoiding the first euro, $\phi'(0)$, is low enough. The results obtained in the previous sections thus carry over to the model with endogenous labor supply.

# 5 Conclusion

We have shown that it is optimal for some individuals to conceal income to the fiscal authority when the avoidance cost is convex. This contrasts with the result of Grochulski (2007), who proves a no-falsification theorem in the case of a subadditive cost function. Our result relies on the idea that permitting avoidance allows to relax incentive constraints as high income individuals are less tempted to mimic lower income ones when these latter avoid taxes. The convexity of the cost function is crucial for this effect to arise and this thus explains the difference in the results between Grochulski (2007) and our setting.

We have assumed a bounded support for the distribution of incomes. A general result in the optimal taxation literature is that, with a bounded support for the distribution of productivities, the marginal tax rate is 0 at the highest sill level (Sadka (1976)). With an unbounded distribution, matters are however different. Diamond (1998) argue that for some utility functions and skill distributions, marginal tax rates may be increasing with productivity and be strictly positive at the limit. It should be noted that in our setting it makes no difference whether the distribution of incomes is bounded or not. One can readily check that all our results go through with an unbounded support. Individuals with income high enough do not conceal income and face a marginal tax rate equal to $\phi'(0)$.

Our results contrast with evidence that points to the fact that the richest taxpayers in society are more prone to enter into tax avoidance activities (Agell and Persson (2000), Roine (2006)). Roine (2006) develops a political economy analysis that offers predictions in line with observed behaviors. He indeed shows that the equilibrium tax rates may be supported by a coalition of the poor and the rich. The poor would like to increase the tax rate because they benefit from the redistribution. The rich are also beneficiaries of the tax system as they exploit the avoidance opportunities and thus end up paying relatively small taxes. The middle-class people do not conceal income and are opposed to a further increase in the tax rate. The equilibrium predictions are thus at odds with the normative recommendation arising out of our model.

The fact that the rich do not conceal income in our normative analysis could be thought to be driven by our assumption that all individuals face the same avoidance opportunities, in the sense that they all face the same cost of avoidance. In section 4.1, we give some arguments why offering to the rich better avoidance opportunities (both with respect to the fixed and the marginal cost of avoidance) would not affect qualitatively the results. These arguments are however derived in an informal way and a more careful analysis is needed.

We have only considered the avoidance response to taxation. In order to get a better sense of the shape of the optimal tax schedule, it is desirable to incorporate in the model real responses to taxation, that is to allow individuals to choose optimally, together with the amount of reported income, their labor supply. We have developed in section 4.2.2 this analysis in the two-types model. The case of a continuum of productivity levels should be addressed in future research.

Finally, we have considered the cost of avoidance as given. This can however be controlled, at least partially, by the government through changes in the tax legislation. A natural extension of this work would consist in endogenizing the cost of avoidance, as Slemrod and Kopczuk (2002) do in the context of a linear income tax.

Appendix

# Proof of proposition 4

Building on (9), the condition for a type $w$ not to avoid is:

$$\mu\phi'_+(0)f(w) + \lambda(w)\phi''_+(0) \geq 0, \tag{16}$$

where $\phi'_+$ and $\phi''_+$ denote the right-hand derivatives.

When $\phi'_+(0) \to 0$, the first term in (16) disappears. Noting that $\lambda$ is negative and $\phi''$ is positive, this condition is violated for any $w \in (w_-, w_+)$. Therefore *it cannot be the case that all individuals declare their true income*. This implies that there exists $\phi_1 \in (0,1)$ such that some individuals avoid taxes as soon as $\phi'_+(0) < \phi_1$.

When $\phi'_+(0) \to 1$, we check that the no-falsification contract characterized in section 3.3 is optimal. In this case, $U(w) \to \overline{w}$ and $\lambda(w) \to 0$ (from (10) and (11)). It is then clear that (16) is satisfied for all $w$, meaning that all individuals declare their true income. This implies that there exists $\phi_2 \in (0,1)$, $\phi_2 \geq \phi_1$, such that no individuals avoids taxes as soon as $\phi'_+(0) \geq \phi_2$.

From (16), no individual will avoid taxes as soon as:

$$\mu\phi'_+(0)f(w) + \lambda(w)\phi''_+(0) \geq 0 \text{ for all } w$$

where

$$\lambda(w) = \int_w^{w_+} (G'((1-\phi'_+(0))t + \overline{w}\phi'_+(0)) - \mu)f(t)dt$$

and

$$\mu = \int G'((1-\phi'_+(0))w + \overline{w}\phi'_+(0))dF(w).$$

This condition is equivalent to

$$-\frac{\lambda(w)}{f(w)} \leq \mu\frac{\phi'_+(0)}{\phi''_+(0)} \text{ for all } w.$$

Therefore $\phi_1$ and $\phi_2$ are values of $\phi'_+(0)$ that solve

$$\max_w -\frac{\lambda(w)}{f(w)} = \mu\frac{\phi'_+(0)}{\phi''_+(0)}.$$

We know from the discussion above that this equation admits at least one solution. When this solution is unique, $\phi_1 = \phi_2$. We cannot however exclude the possibility that this equation has multiple solutions.

We have shown that some individuals will optimally avoid taxation when $\phi'_+(0) < \phi_1$. Inspecting (16) and noting that $\lim_{w \to w_-} \lambda(w) = \lim_{w \to w_+} \lambda(w) = 0$, it appears that individuals close to $w_-$ and $w_+$ should report their true income (except when $\phi'_+(0) = 0$, in which case *only* $w_-$ and $w_+$ do not falsify). There thus exist two threshold values $w_{\text{inf}} \geq w_-$ and $w_{\text{sup}} \leq w_+$ such that individuals with income $w \leq w_{\text{inf}}$ and $w \geq w_{\text{sup}}$ declare their true income. Moreover individuals located closely to the "right" of $w_{\text{inf}}$ and to the "left" of $w_{\text{sup}}$ understate their income report to the fiscal authority; $w_{\text{inf}}$ and $w_{\text{sup}}$ are solutions to

$$\mu \phi'_+(0) f(w) + \lambda(w) \phi''_+(0) = 0. \tag{17}$$

Note that there may exist more than two solutions to this equation, in which case some subsets of individuals located in the interior of the income distribution also declare truthfully.

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